SYLLABUS

Course Title:	**Elements of Electrical Engineering**		
Course Code:	**22EEE13/23**	CIE Marks	50
Course Type (Theory/Practical /Integrated)	Theory	SEE Marks	50
		Total Marks	100
Teaching Hours/Week (L:T:P: S)	2:2:0:0	Exam Hours	03
Total Hours of Pedagogy	40 hours	Credits	03

Course objectives

- To explain the basic laws used in the analysis of DC circuits, electromagnetism.
- To explain the behavior of circuit elements in single-phase circuits.
- To explain three phase circuits, balanced loads and measurement of three phase power.
- To explain the measuring techniques, measuring instruments and domestic wiring.
- To explain electricity billing, equipment and personal safety measures.

Teaching-Learning Process

These are sample Strategies, which teacher can use to accelerate the attainment of the various course outcomes and make Teaching –Learning more effective

1. Chalk and talk 2. Animated/NPTEL videos 3. Cut sections 4. PPTs

Module-1 (08 Hrs)

DC circuits: Ohm's law and Kirchhoff's laws, analysis of series, parallel and series-parallel circuits. Power and energy.

Electromagnetism: Faraday's Laws of Electromagnetic Induction, Lenz's Law, Flemings rules, statically and dynamically induced EMF; concepts of self and mutual inductance. Coefficient of Coupling. Energy stored in magnetic field. Simple Numerical.

Module-2 (08 Hrs)

Single-phase AC circuits: Generation of sinusoidal voltage, frequency of generated voltage, average value, RMS value, form factor and peak factor of sinusoidal voltage and currents.

Phasor representation of alternating quantities. Analysis of R, L, C, R-L, R-C and R-L-C circuits with phasor diagrams, Real power, reactive power, apparent power, and Power factor. Series, Parallel and Series-Parallel circuits. Simple Numerical.

Module-3(08 Hrs)

Three-phase AC circuits: Necessity and advantage of 3-phase system. Generation of 3-phase power. Definition of phase sequence. Balanced supply and balanced load. Relationship between line and phase values of balanced star and delta connections. Power in balanced 3-phase circuits. Measurement of 3-phase power by 2-wattmeter method. Simple Numerical.

Module-4(08 Hrs)

Measuring instruments: construction and working principle of whetstone's bridge, Kelvin's double bridge, Megger, Maxwel's bridge for inductance, Schering's bridge for capacitance, concepts of current transformer and potential transformer.

Domestic Wiring: Requirements, Types of wiring: casing, capping. Two way and three way control of load.

Module-5 (08 Hrs)

Electricity bill: Power rating of household appliances including air conditioners, PCs, laptops, printers, etc. Definition of "unit" used for consumption of electrical energy, two-part electricity tariff, calculation of electricity bill for domestic consumers.

Equipment Safety measures: Working principle of Fuse and Miniature circuit breaker (MCB), merits and demerits.

Personal safety measures: Electric Shock, Earthing and its types, Safety Precautions to avoid shock, and Residual Current Circuit Breaker (RCCB) and Earth Leakage Circuit Breaker (ELCB).

Course outcome (Course Skill Set)

At the end of the course the student will be able to:

CO1	Understand the concepts of DC circuits and Electromagnetism.
CO2	Understand the concepts of single phase and Three phase AC circuits.
CO3	Apply the basic Electrical laws to solve circuits.
CO4	Understand the concepts of measurements and measuring Instruments
CO5	Explain the concepts of domestic wiring, electricity billing, circuit protective devices and personal safety measures.

Assessment Details (both CIE and SEE)

The weightage of Continuous Internal Evaluation (CIE) is 50% and for Semester End Exam (SEE) is 50%. The minimum passing mark for the CIE is 40% of the maximum marks (20 marks out of 50). The minimum passing mark for the SEE is 35% of the maximum marks (18 marks out of 50). A student shall be deemed to have satisfied the academic requirements and earned the credits allotted to each subject/ course if the student secures not less than 35% (18 Marks out of 50) in the semester-end examination(SEE), and a minimum of 40% (40 marks out of 100) in the sum total of the CIE (Continuous Internal Evaluation) and SEE (Semester End Examination) taken together.

Continuous Internal Evaluation(CIE):

Two Unit Tests each of30 Marks (duration 01 hour)

- First test after the completion of 30-40 % of the syllabus
- Second test after completion of 80-90% of the syllabus

One Improvement test before the closing of the academic term may be conducted if necessary. However best two tests out of three shall be taken into consideration

Two assignments each of 20 Marks

The teacher has to plan the assignments and get them completed by the students well before the closing of the term so that marks entry in the examination portal shall be done in time.Formative (Successive) Assessments include Assignments/Quizzes/Seminars/ Course projects/Field surveys/ Case studies/ Hands-on practice (experiments)/Group Discussions/ others.. The Teachers shall choose the types of assignments depending on the requirement of the course and plan to attain the Cos and POs. (to have a less stressed CIE, the portion of the syllabus should not be common /repeated for any of the methods of the CIE. Each method of CIE should have a different syllabus portion of the course). CIE methods /test question paper is designed to attain the different levels of Bloom's taxonomy as per the outcome defined for the course.

The sum of two tests, two assignments, will be out of 100 marks and will be scaled down to 50 marks

Semester End Examination(SEE):

Theory SEE will be conducted by University as per the scheduled timetable, with common question papers for the subject (**duration 03 hours)**

- The question paper shall be set for 100 marks. The medium of the question paper shall be English/Kannada). The duration of SEE is 03 hours.
- The question paper will have 10 questions. Two questions per module. Each question is set for 20 marks.The students have to answer 5 full questions, selecting one full question from each module. The student has to answer for 100 marks and **marks scored out of 100 shall be proportionally reduced to 50 marks**.
- There will be 2 questions from each module. Each of the two questions under a module (with a maximum of 3 sub-questions), **should have a mix of topics** under that module.

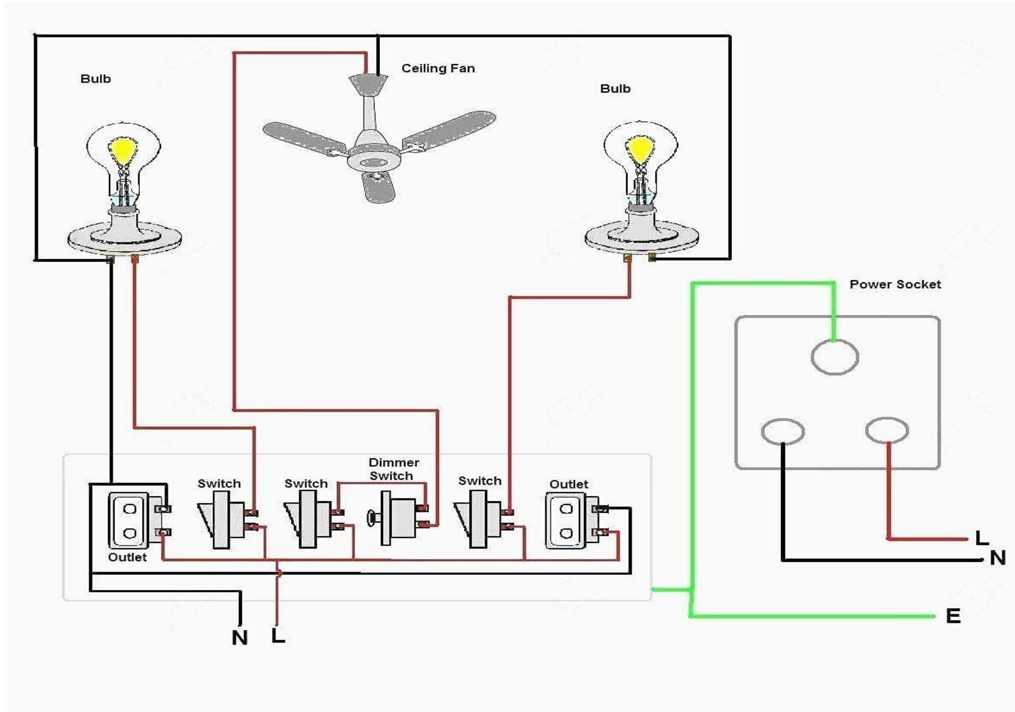
Suggested Learning Resources:

Books (Title of the Book/Name of the author/Name of the publisher/Edition and Year) Text Books:

1. Basic Electrical Engineering by D C Kulshreshtha, Tata McGraw Hill, First Edition 2019.
2. A text book of Electrical Technology by B.L. Theraja, S Chand and Company, reprint edition 2014.

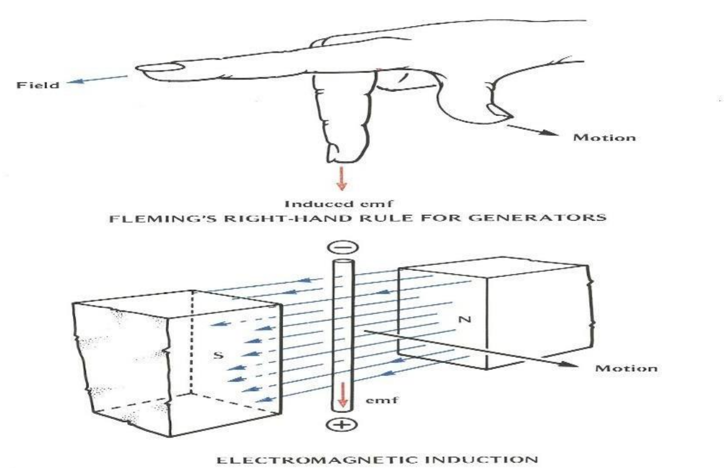
MODULE-2

Single -Phase AC Circuits



2. Generation of sinusoidal AC Voltage: Alternating voltage may be generated:

- By rotating a coil in a magnetic field
- By rotating a magnetic field within a stationary coil



"In each case, the value of the alternating voltage generated depends upon the number of turns in the coil, the strength of the field and the speed at which the coil or magnetic field rotates."

The alternating voltage generated has regular changes in magnitude and direction. If a load resistance (e.g., a light bulb) is connected across this alternating voltage, an alternating current flow in the circuit. When there is a reversal of polarity of the alternating voltage, the direction of current flow in the circuit also reverses.

2.1 Equation of Alternating E.M.F.

Let us take up the case of a rectangular coil of N turns rotating in the anticlockwise direction, with an angular velocity of ω radians per second in a uniform magnetic field as shown below. let the time be measured from the instant of coincidence of the plane of the coil with the X axis. At this instant maximum flux max links with the coil. As the coil rotates, the flux linking with it changes and hence e.m.f. is induced in it. Let the coil turn through an angle θ in time's" seconds, and let it assume the position as shown Obviously $\theta = \omega t$.

When the coil is in this position, the maximum flux acting vertically downwards can be resolved into two components, each perpendicular to the other, namely:

Component $\phi_{max} \sin \omega t$, parallel to the plane of the coil. This component does not induce e.m.f. as it i is parallel to the plane of the coil.

Component $\phi_{max} \cos \omega t$ perpendicular to the plane of coil. This component induces e.m.f. in the coil.

flux linkages of coil at that instant (at θ^0) are

= No. of turns x flux linking

$= N \phi_{max} \cos \omega t$

As per Faraday's Laws of Electromagnetic induction, the e.m.f. induced in a coil is equal to the rate of change of flux linkages of the coil. So, instantaneous e.m.f. „e' induced in the coil at this instant is:

$e = \frac{d}{dt}$ (flux linkages)

$e = \frac{d}{dt} (N \ \phi_{max} \ \cos \omega t)$

$e = - N \phi_{max} \ \frac{d}{dt} \ \phi (\cos \omega t)$

$e = -N \phi_{max} \ \omega (-\sin \omega t)$

$e = + N \phi_{max} \ \sin \phi \omega t$ volts

It is apparent from eqn. (1) that the value of e will be maximum (E_m), when the coil has rotated through

$i = I_m \sin \omega t$

In this case the load is resistive (we shall see, later, that if the load is inductive or capacitive, this current equation is changed in time angle).

90^0 (as $\sin 90^0 = 1$)

Thus $E_m = N \omega \phi_{max}$ volts (2)

Substituting the value of $N \omega \phi_{max}$ from eqn. (2) in eqn. (1), we obtain:

$e = E_m \sin \omega t$ . . .(3)

We know that $\theta = \omega t$

$e = E_m \sin \theta$

∴

It is clear from this expression of alternating e.m.f. induced in the coil that instantaneous emf varies as the sin of the time angle (θ or ωt).

$\omega = 2\pi f$, where „f is the frequency of rotation of the coil. Hence eqn. (3) can be written as

$e = E_m \sin 2\pi f t$ . . . (4)

If T = time of the alternating voltage = $\frac{1}{f}$, then eqn.(iv) may be re-written as

$e = E_m \left(\frac{2\pi}{T}\right) t$

so, the e.m.f. induced varies as the sine function of the time angle, ωt, and if e.m.f. induced is plotted against time, a curve of sine wave shape is obtained as shown in Fig.3.4. Such an e.m.f. is called sinusoidal when the coil moves through an angle of 2π radians.

2.2 Important Definitions

Important terms/definitions, which are frequently used while dealing with a.c. circuits, are as given below:

1. **Alternating quantity**: An alternating quantity is one which acts in alternate positive and negative directions, whose magnitude undergoes a definite series of changes in definite intervals of time and in which the sequence of changes while negative is identical with the sequence of changes while positive.
2. **Waveform**: "The graph between an alternating quantity (voltage or current) and time is called waveform", generally, alternating quantity is depicted along the Y-axis and time along the X-axis.fig.4.4 shows the waveform of a sinusoidal voltage.

3. **Instantaneous value**: The value of an alternating quantity at any instant is called instantaneous value. The instantaneous values of alternating voltages and current are represented by „e' and „I' respectively.

4. **Alternation and cycle**: When an alternating quantity goes through one half cycle (complete set of +ve or –ve values) it completes an alternation, and when it goes through a complete set of +ve and –ve values, it is said to have completed one cycle.

5. **Periodic Time and Frequency**: The time taken in seconds by an alternating quantity to complete one cycle is known as periodic time and is denoted by T.

The number of cycles completed per second by an alternating quantity is known as frequency and is denoted by f. in the SI system, the frequency is expressed in hertz.

The number of cycles completed per second = f.

Periodic Time T – Time taken in completing one cycle = $\frac{1}{f}$ Or f = $\frac{1}{T}$

In India, the standard frequency for power supply is 50 Hz. It means that alternating voltage or current completes 50 cycles in one second.

6. **Amplitude**: The maximum value, positive or negative, which an alternating quantity attains during one complete cycle, is called amplitude or peak value or maximum value. The amplitude of alternating voltage and current is represented by E_m and I_m respectively.

Different Forms of E.M.F. Equation

The standard form of an alternating voltage, as already mentioned and is

$e = E_m \sin\theta = E_m \sin\omega t = E_m \sin 2\pi f t = E_m \sin\frac{2\pi}{T} t$

on perusal of the above equations, we find that

a) The amplitude or peak value or maximum value of an alternating voltage is given by the coefficient of the sine of the time angle.

b) The frequency „f' is given by the coefficient of time divided by 2π.

Taking an example, if the equation is of an alternating voltage is given by e = 20 sin 314t, then its maximum value is 20V and its frequency is

$f = \frac{314}{2\pi}$ 50 Hz

In a like manner if the equation is of the form

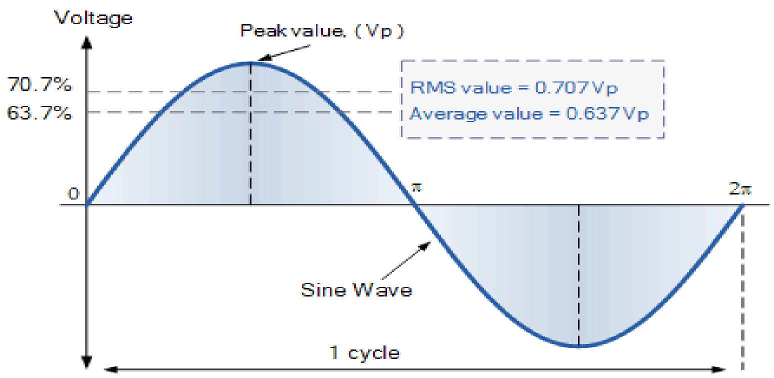
$e = \sqrt{(R^2 + 4\omega^2 L^2)}$ sin 2 ωt, then its maximum value is $E_m = I_m \sqrt{(R^2 + 4\omega^2 L^2)}$ and the frequency is $\frac{2\omega}{2\pi}$ Or $\frac{\omega}{\pi}$ Hertz

2.3 Root-mean-square (R.M.S.) Value:

The r.m.s. or effective value, of an alternating current is defined as that steady current which when flowing through a given resistance for a given time produces the same amount of heat as produced by the alternating current, when flowing through the same resistance for the same time.

Let us take two circuits with identical resistance, but one is connected to a battery and the other to a sinusoidal voltage source. Wattmeters are employed to measure heat power in each circuit. The voltage applied to each circuit is so adjusted that the heat power produced in each circuit is the same. In this event the direct current I will equal $\frac{I_m}{\sqrt{2}}$, which is termed r.m.s. value of the sinusoidal current.

The following method is used for finding the r.m.s. or effective value of sinusoidal waves. The equation of an alternating current varying sinusoid ally is given by i= I_m sin θ.



Let us consider an elementary strip of thickness dθ in the first cycle of the squared wave, as shown.

'Let i^2 be mid-ordinate of this strip.

Area of the strip = i^2 d θ

Area of first half-cycle of squared wave

$= \int_0^{\pi} i^2 \, d\theta$

$= \int_0^{\pi} (I_m \sin\theta)^2 \, d\theta$ (∵ I = I_m sin θ)

=

$\int_0^{\pi} I_m{}^2 \sin^2\theta \, d\theta$

$= I_m{}^2 \int_0^{\pi} \frac{1-\cos 2\theta}{2} \, d\theta$ $= \frac{I_m^2}{2} \int_0^{\pi} (1-\cos 2\theta) d\theta$

$$= \frac{I_m^2}{2}\left[\theta - \frac{sin2\theta}{2}\right]_0^\pi$$

$$= \left[\theta - \frac{sin2\theta}{2}\right]_0^\pi \quad [(\pi - 0) - (0 - 0)]$$

$$= \frac{\pi I_m^2}{2}$$

$(\because \sin^2 \quad =)$

$$\sqrt{\frac{\text{Area of first half cycle of squared wav}}{\text{base}}}$$

$$= \sqrt{\frac{\pi I_m^2}{2} \times \frac{1}{\pi}}$$

$= \sqrt{\frac{I_m^2}{2}} = 0.707 I_m$

Hence, for a sinusoidal current,

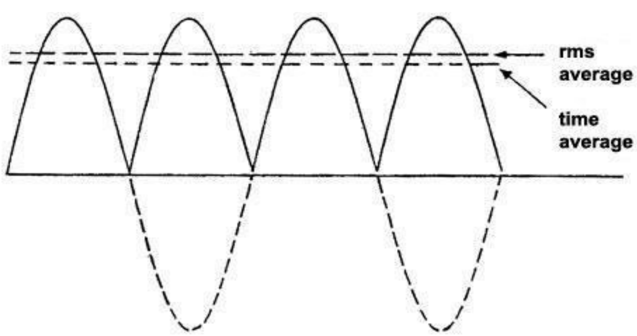
R.M.S. value of current = 0.707 x maximum value of current. Similarly, E =
0.707 E_m

2.4 Average Value

The arithmetical average of all the values of an alternating quantity over one cycle is called average value. In the case of a symmetrical wave e.g., sinusoidal current or voltage wave, the positive half is exactly equal to the negative half, so that the average value over the entire cycle is zero. Hence, in this case, the average value is obtained by adding or integrating the instantaneous values of current over one alternation (half-cycle) only.

The equation of a sinusoidally varying voltage is given by $e = E_m \sin\theta$.

Let us take an elementary strip of thickness $d\theta$ in the first half-cycle as shown. Let the mid-ordinate of this strip be e.



Area of the strip = e. $d\theta$

Area of first half-cycle

$= \int_0^{\pi} e \, d\theta$

$= \int_0^{\pi} E_m \sin\theta \, d\theta$ $\quad (\because e = E_m \sin\theta)$

$= E_m \int_0^{\pi} \sin\theta \, d\theta$

$= E_m [-\cos\theta]_0^{\pi} = 2E_m$

Average value, $E_{av} = \frac{\text{Area of half cycle}}{\text{base}} = \frac{2E_m}{\pi}$ Or $E_{av} = 0.637\,E_m$

$\therefore$ In a similar manner, we can prove that, for alternating current varying sinusoidally, $I_{av} = 0.637 I_m$

Average value of current = 0.637 x maximum value

2.5 Form Factor and crest or peak or Amplitude Factor (Kf)

A definite relationship exists between crest value (or peak value), average value and r.m.s. value of an alternating quantity.

Form Factor: The ratio of effective value (or r.m.s. value) to average value of an alternating quantity (voltage or current) is called form factor, i.e.

From Factor, $K_f = \frac{\text{rms value}}{\text{average value}}$

For sinusoidal alternating current,

$K_f = \frac{0.707 I_m}{0.637 I_m} = 1.11$

For sinusoidal alternating voltage,

$K_f = \frac{0.707 I_m}{0.637 I_m} = 1.11$

Hence, the R.M.S. value (of current or voltage) is 1.11 times its average value.

***2.6* Crest or Peak or Amplitude Factor (Ka):** It is defined as the ratio of maximum value to the effective value (r.m.s. value) of an alternating quantity. i.e.,

$K_a = \frac{\text{maximum value}}{\text{r.m.s.value}}$ For sinusoidal alternating current,

$K_a = \frac{I_m}{\frac{I_m}{\sqrt{2}}} == 1.414 = \sqrt{2}$

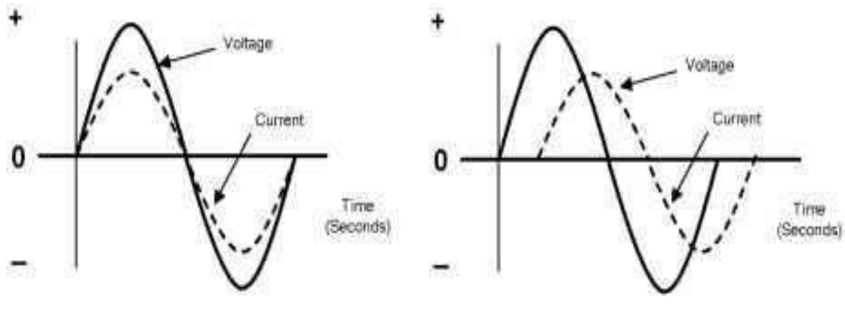
For sinusoidal alternating voltage,

$K_a = \frac{E_m}{\frac{E_m}{\sqrt{2}}} = 1.414$

The knowledge of Crest Factor is particularly important in the testing of dielectric strength of insulating materials; this is because the breakdown of insulating materials depends upon the maximum value of voltage.

2.7 **Phase**

An alternating voltage or current changes in magnitude and direction at every instant. So, it is necessary to know the condition of the alternating quantity at a particular instant. The location of the condition of the alternating quantity at any instant is called its phase.



In Phase Voltage and Current generated by the utility

Out-of-phase Phase Voltage and Current caused by use of inductors

2.8. Phase Difference (Lagging or Leading of Sinusoidal wave)

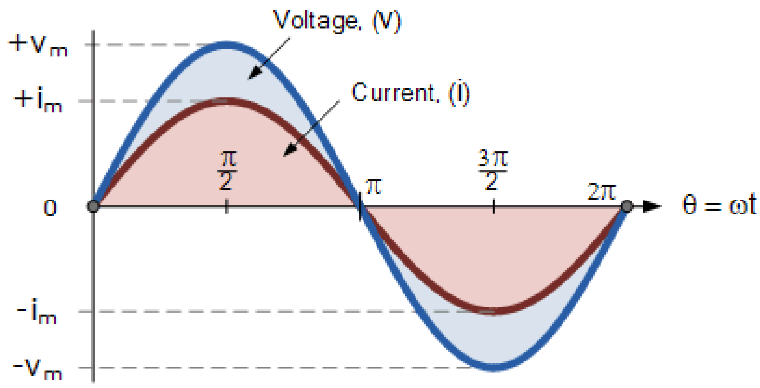
When two alternating quantities, say, two voltages or two currents or one voltage and one current are considered simultaneously, the frequency being the same, they may not pass through a particular point at the same instant.

One may pass through its maximum value at the instant when the other passes through a value other than its maximum one. These two quantities are said to have a phase difference. Phase difference is specified either in degrees or in radians.

The phase difference is measured by the angular difference between the points where the two curves cross the base or reference line in the same direction. The quantity ahead in phase is said to lead the other.

1. **Phase**

   An alternating voltage or current changes in magnitude and direction at every instant. So, it is



necessary to know the condition of the alternating quantity at a particular instant. The location of the condition of the alternating quantity at any particular instant is called its phase.

We may define the phase of an alternating quantity at any particular instant as the fractional part of a period or cycle through which the quantity has advanced from the selected origin.
Taking an example, the phase of current at point A (+ve maximum value) is T/4 second, where T is the time period, or expressed in terms of angle, it is π/2 radians (Fig.3.7). In other words, it means that the condition of the wave, after having advanced through π/2 radians (900) from the selected origin (i.e.,0) is that it is maximum value (in the positive direction).similarly, -ve maximum value is reached after 3π/2 radians (2700) from the origin, and the phase of the current at point B is 3T/4 second.

2. Phase Difference (Lagging or Leading of Sinusoidal wave)

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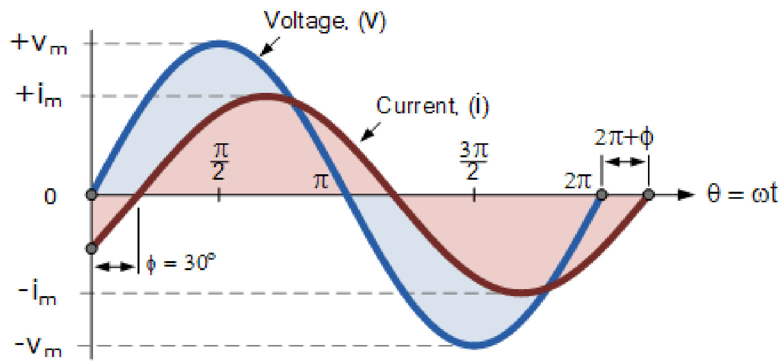
The phase difference is measured by the angular difference between the points where the two curves cross the base or reference line in the same direction.

The quantity ahead in phase is said to lead the other quantity, whereas the second quantity is said to lag behind the first one. In Fig.3.8, current I1, represented by vector 0A, leads the current I2, represented by vector 0B, by ϕ or Current I2 lags behind the current I1 by ϕ

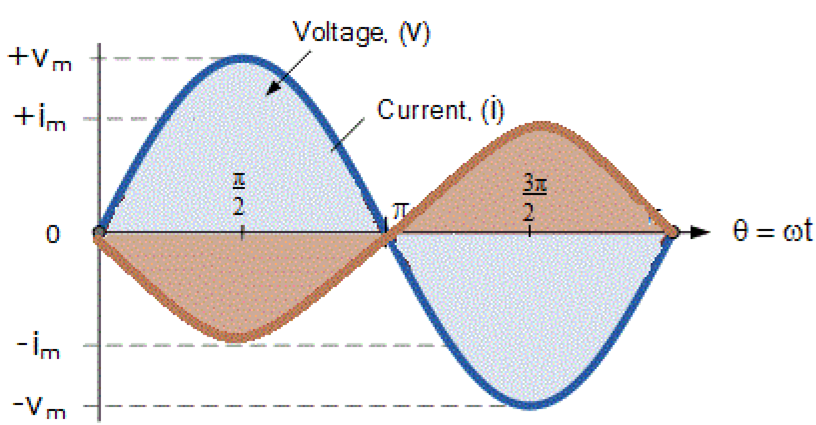


The leading current I1 goes through its zero and maximum values first and the current I2 goes through its zero and maximum values after time angle ϕ. The two waves representing these two currents are shown in Fig.3.8. if I1 is taken as reference vector, two currents are expressed as

$i_1 = I_{1m} \sin \omega t$

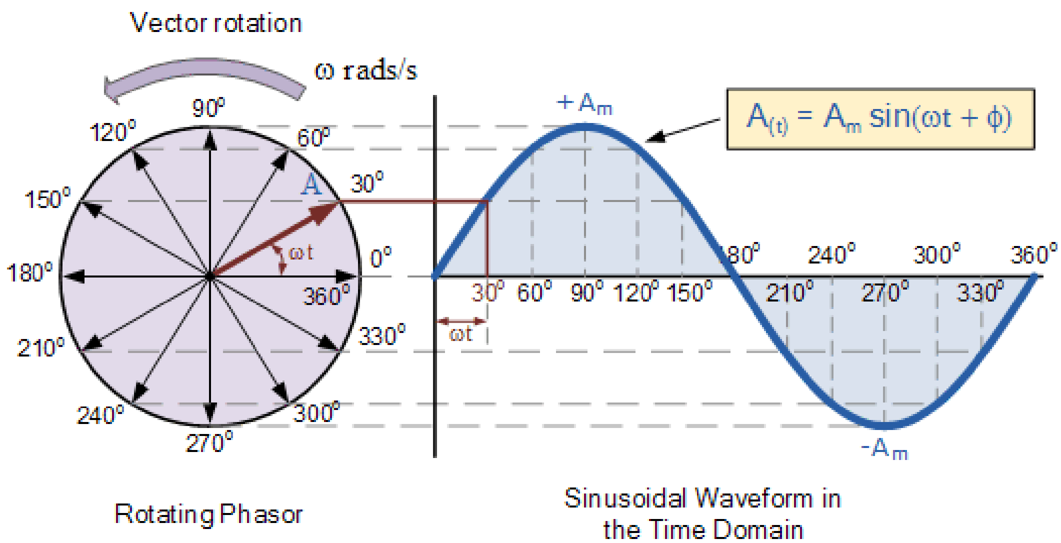
$i_2 = I_{2m} \sin (\omega t - 120^0)$

The two quantities are said to be in phase with each other if they pass through zero values at the same instant and rise in the same direction, as shown in Fig.3.9. However, if the two quantities pass through zero values at the same instant but rise in opposite, as shown in Fig.3.10, they are said to be in phase opposition i.e., the phase difference is 180^0. When the two alternating quantities have a phase difference of 90^0 or $\pi/2$ radians they are said to be in quadrature.



3. **Phasor Representation of Alternating Quantities**

We know that an alternating voltage or current has sine waveform, and generators are designed to give e.m.f.s. with the sine waveforms. The method of representing alternating quantities continuously by equation giving instantaneous values (like e = Em sin ωt) is quite tedious. So, it is more convenient to represent a sinusoidal quantity by a phasor rotating in an anticlockwise direction.



While representing an alternating quantity by a phasor, the following points are to be kept in mind:

i) The length of the phasor should be equal to the maximum value of the alternating quantity.

ii) The phasor should be in the horizontal position at the alternating quantity is zero and is increasing in the positive direction.

iii) The inclination of the line with respect to some axis of reference gives the direction of that quantity and an arrow-head placed at one end indicates the direction in which that quantity acts.

iv) The angular velocity in an anti-clockwise direction of the phasor should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle.Consider phasor 0A, which represents the maximum value of the alternating e.m.f. and its angle with the horizontal axis gives its phase (Fig.3.12). now, it will be seen that the projection of this phasor 0A on the vertical axis will give the instantaneous value of e.m.f.

$\therefore$ 0B = 0A sin ωt

Or e = 0A sin ωt

= Em sin ωt

Note: Theterm“phasor” isalso known “vector”.

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a) 8+j6 = $\sqrt{8^2 + 6^2}$ ∟tan^{-1} 0.75=10∟36.9^0

b) -10-j7.5=∟tan^{-1} 0.75=12.5 ∟36.9^0

This vector also falls in the third quadrant, so, following the same reasoning as mentioned in method 1, the angle when measured in CCW direction is
= tan^{-1} 0.75

= 180^0+36.9^0=216.9^0

Measured in CCW direct from + ve co-ordinate of x-axis, the angle is -(360^0 – 216.9^0) = - 143.1^0
So this expression is written as 12.5 ∟-143.1^0 So, expression (ii) is rewritten as
10∟ 36.9^0×12.5 ∟-143.1^0

125-106.2^0 which is the same as before.

4. Analysis of A.C circuits

The path for the flow of alternating current is called on a.c. circuit.

In a d.c. circuit, the current/flowing through the circuit is given by the simple relation I = $\frac{V}{R}$. However, in an a.c. circuit, voltage and current change from instant to instant and so give rise to magnetic (inductive) and electrostatic (capacitive) effects. So, in an a.c. circuit, inductance and capacitance must be considered in addition to resistance.

We shall now deal with the following a.c. circuits:

i) AC circuit containing pure ohmic resistance only.

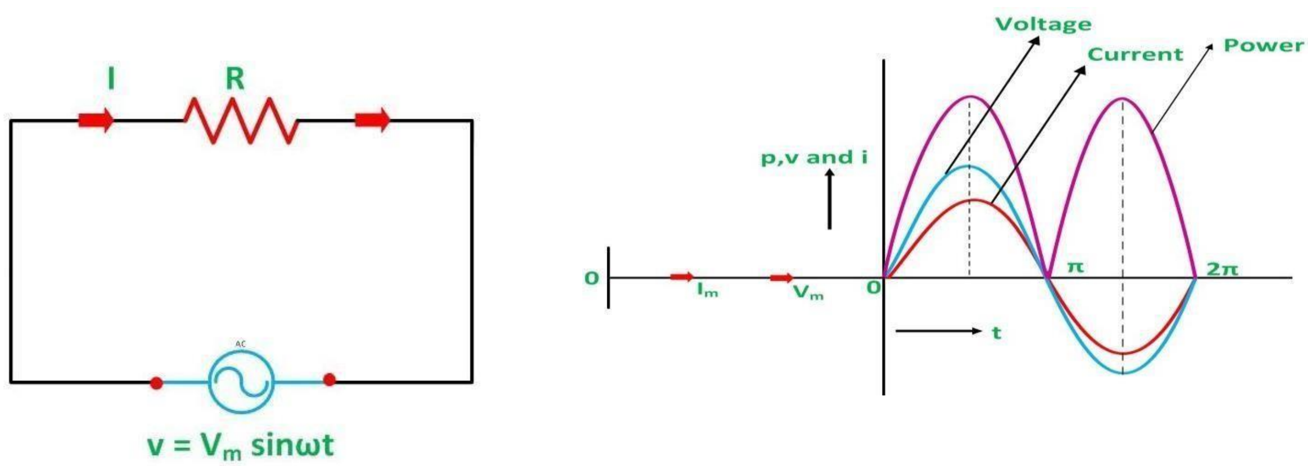
ii) AC circuit containing pure inductance only.

iii) AC circuit containing pure capacitance only.

5. AC circuit containing pure ohmic Resistance

When an alternating voltage is applied across a pure ohmic resistance, electrons (current) flow in one direction during the first half-cycle and in the opposite direction during the next half- cycle, thus constituting alternating current in the circuit.

Let us consider an a.c. circuit with just a pure resistance R only, as shown in Fig.3.31.



Let the applied voltage be given by the equation

$V = V_m \sin \theta = V_m \sin$ .... (i)

As a result of this alternating voltage, alternating current „i" will flow through the circuit. The applied voltage has to supply the drop in the resistance, i.e.,

$V = IR$

Substituting the value of "V" from eqn. (i), we get

$V_m \sin \omega t = IR$ or $I = \frac{V_m}{R} \sin \omega t$ .... (ii)

The value of the alternating current I is maximum when $\sin \omega t = 1$,

i.e., $I_m = \frac{V_m}{R}$

Eqn.(ii) becomes,

$i = I_m \sin \omega t$ .... - (iii)

From eqns.(i) and (ii), it is apparent that voltage and current are in phase with each other. This is also indicated by the wave and vector diagram shown in Fig. 3.32.

Power: The voltage and current are changing at every instant.

$\therefore$ Instantaneous power, $P = V_m \sin\omega t = I_m \sin \omega t$

$$V_m I_m \sin^2 \omega t$$

$$= V_m I_m \frac{(1-\cos 2\omega t)}{2}$$

$$= . \cos 2 \quad \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \quad \omega t$$

Thus instantaneous power consists of a constant part $\frac{V_m I_m}{2}$ and a

Fluctuating part $\frac{V_m I_m}{2} \cos 2\omega t$ of frequency double that of voltage and current waves.

The average value of $\frac{V_m I_m}{2} \cos 2\omega t$ over a complete cycle is zero.

So, power for the complete cycle is

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

or $\quad P = V1$ watts

Where $\quad V$ = r.m.s. value of applied voltage

$\quad I$ = r.m.s. value of the current

6. Power curve

The power curve for a purely resistive circuit is shown in Fig. 3.33. It is apparent that power in such a circuit is zero only at the instants a,b and c, when both voltage and current are zero, but is positive at all other instants. in other words, power is never negative, so that power is always lost in a resistive a.c. circuit. This power is dissipated as heat.

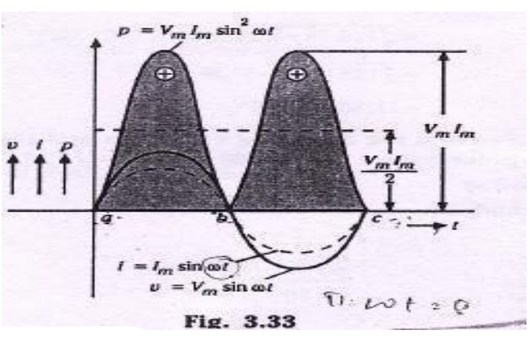


Fig. 3.33

7. <u>A.C. circuit containing pure Inductance</u>

An inductive coil is a coil with or without an iron core and has negligible resistance. In practice, pure inductance can never be had as the inductive coil has always a small resistance. However, a coil of thick copper wire wound on a laminated iron core has negligible resistance, so, for the purpose of our study, we will consider a purely inductive coil.

On the application of an alternating voltage (Fig.3.34) to a circuit containing a pure inductance, a back e.m.f. is produced due to the self-inductance of the coil. This back e.m.f. opposes the rise or fall o f current, at every stage. Because of the absence of voltage drop, the applied voltage has to overcome this self-induced e.m.f. only.

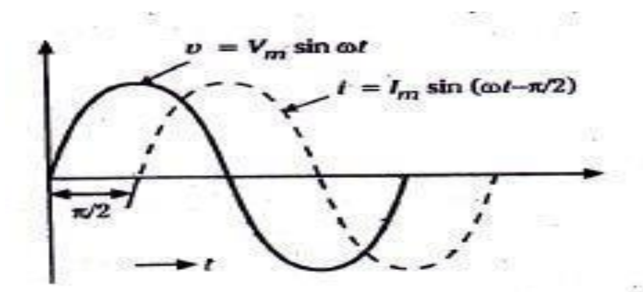


Fig. 3.35

Inductive Reactance: ωL in the expression $I_m = \frac{V_m}{\omega L}$ is known as inductive reactance and is denoted by X_L, i.e., $X_L = \omega L$. If „L' is in henry and „ ω " is in radians per second, then X_L will be in ohms. So, inductive reactance plays the part the part of resistance.

Power: Instantaneous Power,

$$P = v \times i = V_m \sin \omega t \cdot I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -V_m I_m \sin \omega t \cos \omega t$$

$$= \frac{-V_m I_m}{2} \sin 2\omega t$$

The power measured by a wattmeter is the average value of „p", which is zero since average of a sinusoidal quantity of double frequency over a complete cycle is zero. Put in mathematical terms, Power for the whole cycle, P = - $\frac{-V_m I_m}{2}$ $2\omega t$

$\int_0^{2\pi} \sin$ dt = 0 Hence, power absorbed in a pure inductive circuit is zero.

Power curve

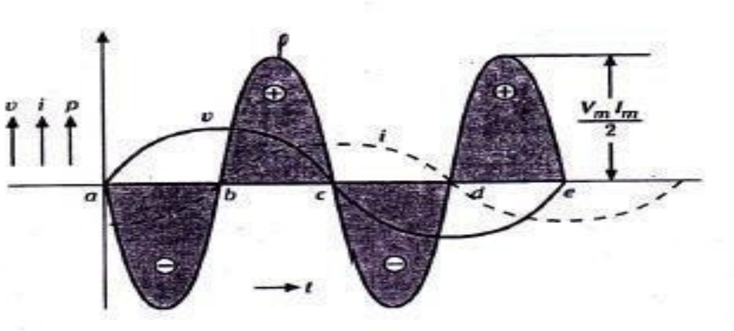


Fig. 3.36

The power curve for a pure inductive circuit is shown in Fig. 3.36. This indicates that power absorbed in the circuit is zero. At the instants a,c and e, voltage is zero, so that power is zero: it is also zero at points b and d when the current is zero. Between a and b voltage and current are in opposite directions, so that power is negative and energy is taken from the circuit. Between b and c voltage and current are in the same direction, so that power is positive and is put back into the circuit. Similarly, between c and d, power is taken from the circuit and between d and e it is put into the circuit. Hence, net power is zero.

8. AC circuit containing pure capacitance

When an alternating voltage is applied across the plates of a capacitor, the capacitor is charged in one direction and then in the opposite direction as the voltage reverses. With reference to Fig. 3.38,

$v \quad V_m \sin \omega t$

Let alternating voltage represented by capacitance C Farads. be applied across a capacitor of v ωt

Instantaneous charge, q = c = CV$_m$ sin

Capacitor current is equal to the rate of change of charge, or

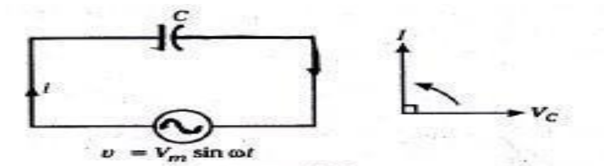


Fig. 3.38

$i = \frac{dq}{dt} = \frac{d}{dt}(CV_m \sin\ )$

$\omega \qquad = \omega t CV_m \cos$

or $\frac{V_m}{\frac{1}{\omega c}}$ sin ωt

The current is maximum when t = 0

$\therefore \qquad I_m \frac{V_m}{\frac{1}{\omega c}}$

Substituting $\frac{V_m}{\frac{1}{\omega c}} = I_m$ in the above expression for instantaneous current, we get $i = I_m \sin\ \omega t$

Capacitive Reactance: $\frac{1}{\omega c}$ in the expression $I_m \frac{V_m}{\frac{1}{\omega c}}$ is known as capacitive reactance and is denoted by X_C.

i.e., $X_C = \frac{1}{\omega c}$

If C is farads and „ ω " is in radians, then X_C will be in ohms.

It is seen that if the applied voltage is given by $v = V_m \sin \omega t$, then the current is given by i

$= I_m \sin\ \omega t$ this shows that the current in a pure capacitor leads its voltage by a

quarter cycle as shown in Fig. 3.39, or phase difference between its voltage and current

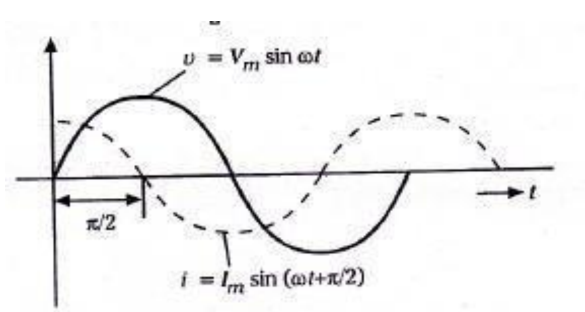


Fig. 3.39

is $\frac{\pi}{2}$ with the current leading.

Power: Instantaneous Power, P = vi

$= V_m \sin \omega t . I_m \sin$

$= V_m \quad \omega$

Power for the complete cycle $\frac{1}{2} V_m$

$I_m \int_0^{2\pi} \sin 2\omega t \ dt = 0 \frac{1}{2} V_m$

Hence power absorbed in a capacitive circuit is zero

At the instants b,d, the current is zero, so that power is zero; it is also zero at the instants a,c and e, when the voltage is zero. Between a and b, voltage and current are in the same direction, so that power is positive and is being put back in the circuit. Between b and c, voltage and current are in the opposite directions, so that power is negative and energy is taken from the circuit. Similarly, between c and d, power is put back into the circuit, and between d and e it is taken from the circuit.

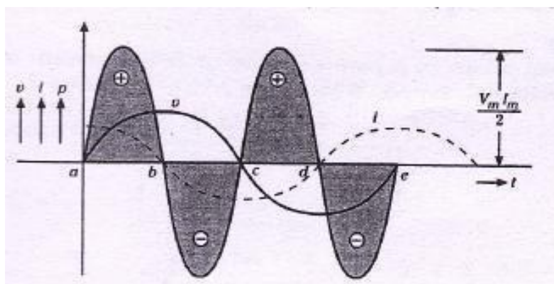


Fig. 3.40

Therefore, power absorbed in a pure capacitive circuit is zero.

Series R-L circuit

Let us consider an a.c. circuit containing a pure resistance R ohms and a pure inductance of L henrys, as shown in Fig. 3.43.

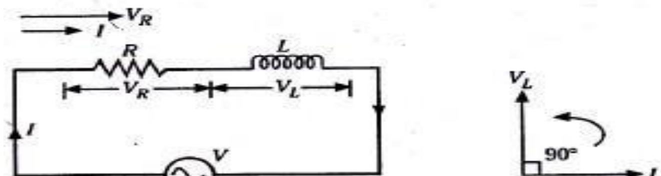


Fig. 3.43

Let V = r.m.s. value of the applied voltage

I = r.m.s. value of the current

Voltage drop across R, $V_R = IR$ (in phase with I)

Voltage drop across L, $V_L = IX_L$ (leading I by 90^0)

The voltage drops across these two circuit components are shown in Fig. 3.44, where vector OA indicates V_R and AB indicates V_L. The applied voltage V is the vector sum of the two, i.e., OB.

$$\therefore \quad V=\sqrt{V_R^2+V_L^2}=\sqrt{(IR)^2+(IX_L)^2}$$

$$= I\sqrt{R^2+X_L^2}$$

$$\therefore \quad I=\frac{V}{\sqrt{R^2+X_L^2}}$$

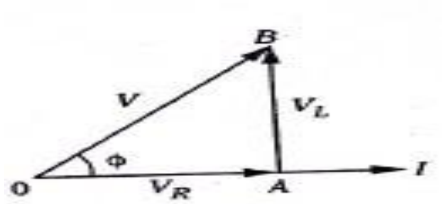


Fig. 3.44

The term $\sqrt{R^2+X_L^2}$ offers opposition to current flow and is called the impedance (Z) of the circuit. It is measured in ohms.

$$\therefore \quad I=\frac{V}{Z}$$

Referring to the impedance triangle ABC, (Fig. 3.45)

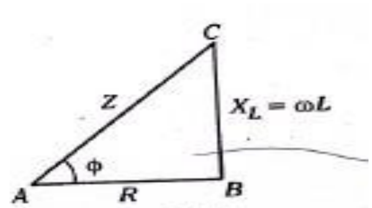


Fig. 3.45

$$Z^2 = R^2 + X_L^2$$

or (impedance)^2 + (reactance)^2

Referring back to Fig. 3.44, we observe that the applied voltage V leads the current I by an angle ϕ.

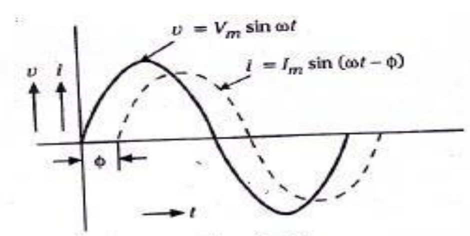


Fig. 3.46

$$\tan\phi=\frac{V_L}{V_R}=\frac{I.X_L}{I.R}=\frac{X_L}{R}=\frac{\omega L}{R}=\frac{\text{reactance}}{\text{resistance}}$$

$$\therefore \quad \phi=\tan^{-1}\frac{X_L}{R}$$

The same feature is shown by means of waveforms (Fig. 3.46). We observe that circuit current lags behind applied voltage by an angle ϕ.

So, if applied voltage is expressed as $v = V_m \sin \omega t$, the current is given by $i = I_m(\sin \omega t - \phi)$,

Where $I_m = \frac{V_m}{Z}$.

Definition of Real power, Reactive Power, Apparent power and power Factor

Let a series R-L circuit draw a current I (r.m.s. value) when an alternating voltage of r.m.s. value V is applied to it. Suppose the current lags behind the applied voltage by an angle ϕ as shown in Fig. 3.47.

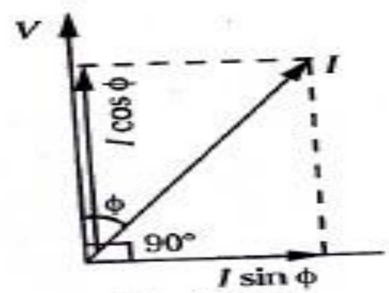


Fig. 3.47

Power Factor and its signifies

Power Factor may be defined as the cosine of the angle of lead or lag. In Fig. 3.47, the angle of lag is shown. Thus power Factor = cos ϕ

In addition to having a numerical value, the power factor of a circuit carries a notation that signifies the nature of the circuit, i.e., whether the equivalent circuit is resistive, inductive or capacitive. Thus, the p.f. might be expressed as 0.8 lagging. The lagging and leading refers to the phase of the current vector with respect to the voltage vector. Thus, a lagging power factor

means that the current lags the voltage and the circuit is inductive in nature. However, in the case of leading power factor, the current leads the voltage and the circuit is capacitive.

Apparent Power: The product of r.m.s. values of current and voltage, VI, is called the apparent power and is measured in volt-amperes (VA) or in kilo-volt amperes (KVA).

Real Power: The real power in an a.c. circuit is obtained by multiplying the apparent power by the factor and is expressed in watts or killo-watts (kW).

Real power (W) = volt-amperes (VA) × power factor cos ϕ

or **Watts = VA cos ϕ**

Here, it should be noted that power consumed is due to ohmic resistance only as a pure inductance does not consume any power.

Thus, $P = V I \cos\phi$

$\cos\phi = \frac{R}{Z}$ (refer to the impedance triangle of Fig. 3.45)

$\therefore \; P = V I \times \left[\frac{R}{Z}\right]$

$= \left[\frac{V}{Z}\right] \times IR = I^2R$

or $P = I^2R$ *watts*

Reactive Power: It is the power developed in the inductive reactance of the circuit. The quantity VI sin ϕ is called the reactive power; it is measured in reactive volt-amperes or vars (VAr).

The power consumed can be represented by means of waveform in Fig. 3.48.

We will now calculate power in terms of instantaneous values.

Instantaneous power, $P = vi = V_m \sin \omega t \times I_m \sin(\omega t - \phi)$

$= V_m I_m \sin \omega t \sin(\omega t - \phi)$

$= \frac{1}{2} V_m I_m [\cos\phi - \cos(2\omega t - \phi)]$

This power consists of two parts:

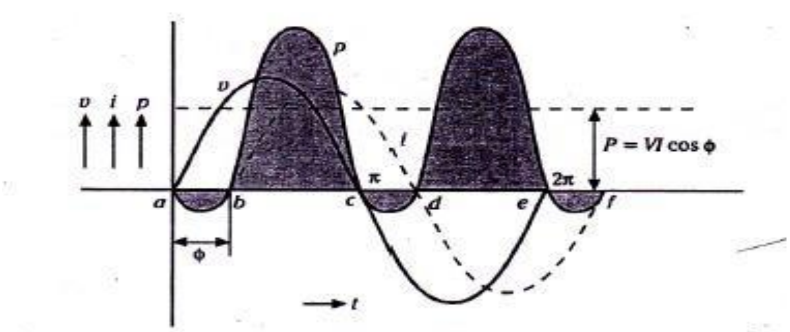


Fig. 3.48

i) Constant part $\frac{1}{2} V_m I_m \cos\phi$ which contributes to real power.

ii) Sinusoidally varying part $\frac{1}{2} V_m I_m \cos(2\omega t - \phi)$, whose frequency is twice that of the voltage and the current, and whose average value over a complete cycle is zero (so it does not contribute to any power).

So, average power consumed, $P = \frac{1}{2} V_m I_m \cos\phi$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \phi$$

$$= V\, I \cos \phi$$

Where V and I are r.m.s. values

Power curves:

The power curve for R-L series circuit is shown in Fig. 3.48. The curve indicates that the greater part is positive and the smaller part is negative, so that the net power over the cycle is positive.

During the time interval a to b, applied voltage and current are in opposite directions, so that power is negative. Under such conditions, the inductance L returns power to the circuit. During the period b to c, the applied voltage and current are in the same direction so that power is positive , and therefore, power is put into the circuit. In a similar way, during the period c to d, inductance L returns power to the circuit while between d and e, power is put into the circuit. The power absorbed by resistance R is converted into heat and not returned.

Series R – C circuit

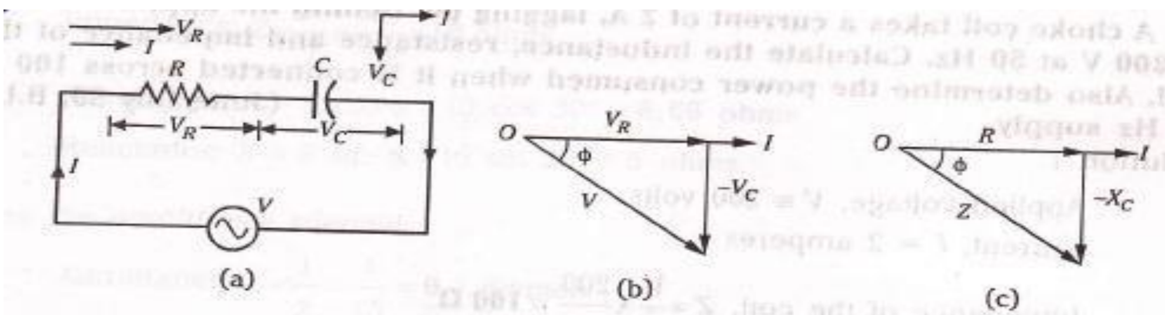


Fig. 3.52

Consider an a.c. circuit containing resistance R ohms and capacitance C farads, as shown in the fig. 3.52(a).

Let V = r.m.s. value of voltage

I = r.m.s. value of current

∴ voltage drop across R, $V_R = IR$ - in phase with I

Voltage drop across C, $V_C = IX_C$ - lagging I by $\frac{\pi}{2}$

The capacitive resistance is negative, so V_C is in the negative direction of Y – axis, as shown in the fig. 3.52(b).

We have V = $\sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (-IX_C)^2}$

= $\sqrt{R^2 + X_C{}^2}$

Or I = $\frac{V}{\sqrt{R^2+X_C{}^2}} = \frac{V}{Z}$

The denominator, Z is the impedance of the circuit, i.e., Z = $\sqrt{R^2 + X_C{}^2}$. fig. 3.52(c) depicts the impedance triangle.

Power factor, $\cos\phi = \frac{R}{Z}$

Fig. 3.52(b) shows that I leads V by an

angle ϕ, so that $\tan\phi = \frac{-X_C}{R}$

This implies that if the alternating voltage is $v = V_m \sin\omega t$, the resultant current in the R – C circuit is given by

$i = I_m \sin(\omega t + \phi)$ such that current leads the applied voltage by the angle ϕ. The waveforms of fig. 3.53 depict this.

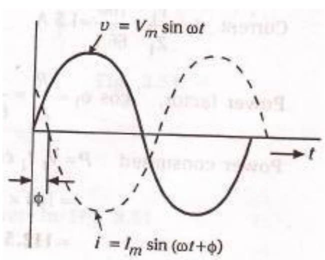


Fig. 3.53

Power: Average power, P = v × I = VI cos ϕ (as in sec. 3.17).

Power curves: The power curve for R – C series circuit is shown in fig. 3.54. The curve indicates that the greater part is positive and the smaller part is negative, so that the net power is positive.

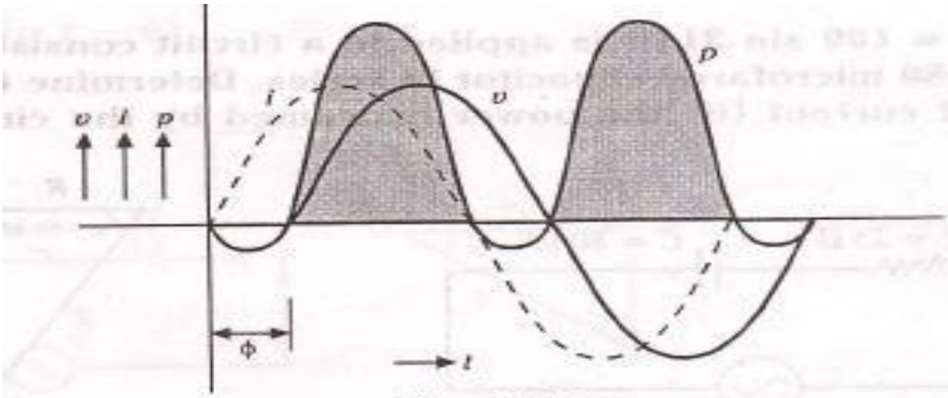


Fig. 3.54

Resistance, Inductance and capacitance in series (RLC – Series Circuit)

Consider an a.c. series circuit containing resistance R ohms, Inductance L henries and

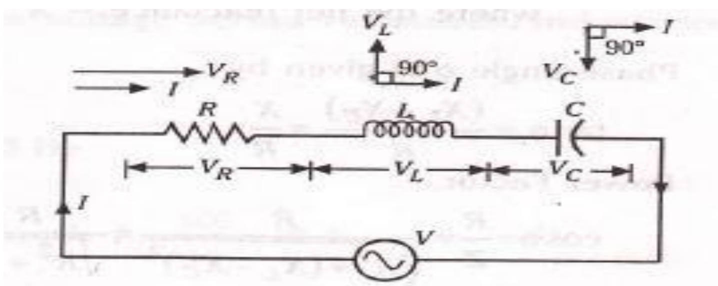


Fig. 3.59

capacitance C farads, as shown in the fig. 3.59.

Let V = r.m.s. value of applied voltage

I = r.m.s. value of current

∴ Voltage drop across R, VR = IR - in phase with I

voltage drop across L, VL = I.X_L - lagging I by 90^0

Voltage drop across C, $V_C = IX_C$ - lagging I by 90^0

Referring to the voltage triangle of Fig. 3.60, OA represents V_R, AB and AC represent inductive and capacitive drops respectively. We observe that V_L and V_C are 180^0 out of phase.

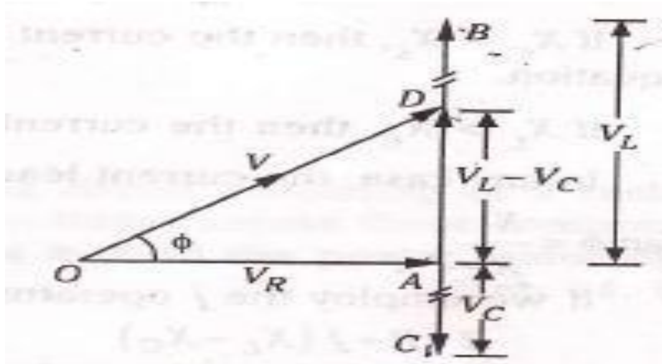


Fig. 3.60

Thus, the net reactive drop across the combination is

$$AD = AB - AC$$

$$= AB - BD \quad (\because BD = AC)$$

$$= V_L - V_C$$

$$= \mathbf{I}\,(X_L - X_C)$$

OD, which represents the applied voltage V, is the vector sum of OA and AD.

$$\therefore \quad OD = \sqrt{OA^2 + AD^2} \qquad \text{OR } V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\text{Or } I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + X^2}} = \frac{V}{Z}$$

The denominator $\sqrt{R^2 + (X_L - X_C)^2}$ is the impendence of the circuit.

So $(\text{impedance})^2 = (\text{resistance})^2 + (\text{net reactance})^2$

$$\text{Or } Z^2 = R^2 + (X_L - X_C)^2 = R^2 + X^2$$

Where the net reactance = X (fig. 3.61)

Phase angle ϕ is given by

$$\tan\phi = \frac{(X_L - X_C)}{R} = \frac{X}{R}$$

power factor,

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2+(X_L-X_C)^2}} = \frac{R}{\sqrt{R^2+X^2}}$$

Power = VI cos ϕ

If applied voltage is represented by the equation $v = V_m \sin\omega t$, then the resulting current in an R – L – C circuit is given by the equation

$$i = I_m \sin(\omega t \pm \phi)$$

If $X_C > X_L$, then the current leads and the +ve sign is to be used in the above equation.

If $X_L > X_C$, then the current lags and the –ve sign is to be used.

If any case, the current leads or lags the supply voltage by an angle ϕ, so that $\tan\phi = \frac{X}{R}$

If we employ the j operator (fig. 3.62), then we have

$$Z = R + j(X_L - X_C)$$

The value of the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The phase angle $\phi = \tan^{-1}\frac{(X_L-X_C)}{R}$

$$Z\angle\phi = Z\angle\tan^{-1}\left[\frac{X_L-X_C}{R}\right]$$

$$= Z\angle\tan^{-1}\left[\frac{X}{R}\right]$$

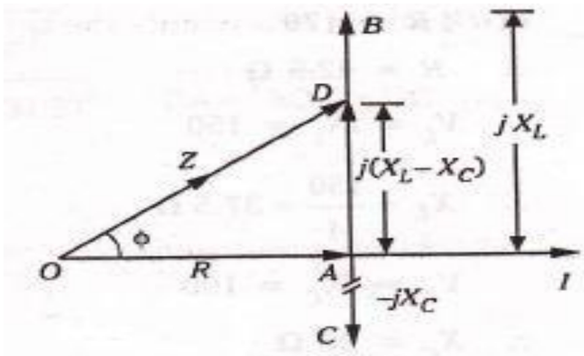


Fig. 3.62

Parallel AC circuits

In a parallel a.c. circuit, the voltage across each branch of the circuit is the same whereas current in each branch depends upon the branch impedance. Since alternating currents are vector quantities, total line current is the vector sum of branch currents.

The following are the three methods of solving parallel a.c. circuits:

a) Vector method.
b) Admittance method.

c) Symbolic or j- method.

3.20.1 Vector method

In this method the total line current is found by drawing the vector diagram of the circuit. As voltage is common, it is taken as the reference vector and the various branch currents are represented vectorially. The total line current can be determined from the vector diagram either by the parallelogram method or by the method of components.

Branch 1

Impedance $Z_1 = \sqrt{R_1^2 + X_L^2}$

Current $I_1 = \frac{V}{Z_1}$

$\cos \phi_1 = \frac{R_1}{Z_1}$ or $\phi_1 = \cos^{-1}\left(\frac{R_1}{Z_1}\right)$

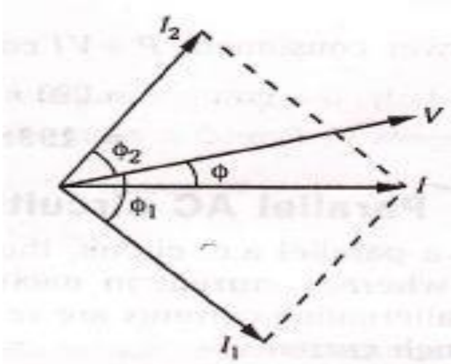


Fig. 3.65

Current I_1 lags behind the applied voltage by ϕ (fig. 3.65).

Branch 2

Impedance $Z_2 = \sqrt{R_2^2 + X_C^2}$

Current $I_2 = \frac{V}{Z_2}$

$\cos \phi_2 = \frac{R_2}{Z_2}$ or $\phi_2 = \cos^{-1}\left(\frac{R_2}{Z_2}\right)$

Current I_2 leads V by ϕ_2 (fig. 3.65).

Resultant current : The total line current I is the vector sum of the branch currents I_1 and I_2 and is found by using the parallelogram law of vectors, as shown in fig. 3.65.

The second method is the method of components i.e., resolving the branch currents I_1 and I_2 along the x- axis and y- axis and then finding the resultant of these components (fig. 3.66).

Let the resultant current be I and ϕ be its phase angle, as shown in fig. 3.66 (b). Then the components of I along X- axis is equal to the algebraic sum of the components of branch currents I_1 and I_2 along the X- axis (active components).

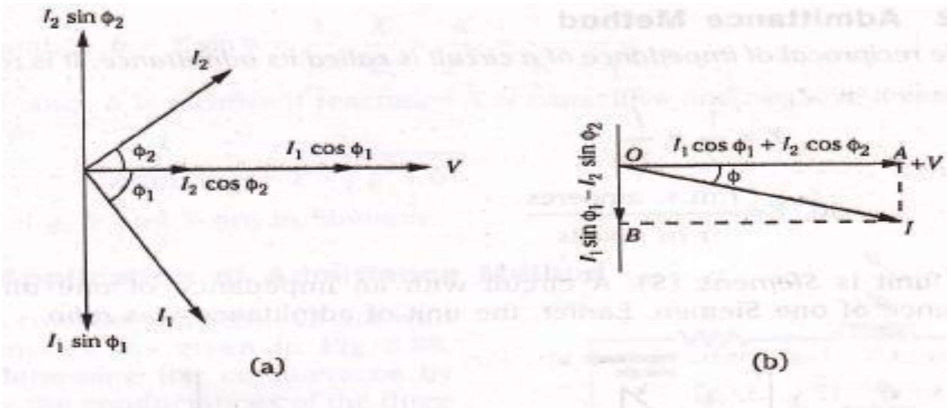


Fig. 3.66

Similarly, the component of I along Y- axis is equal to the algebraic sum of the components of I_1 and I_2 along Y- axis i.e,

Component of resultant current along Y- axis

= algebraic sum of I_1 and I_2 along X – axis

or $I \cos \phi = I_1 \cos \phi_1 + I_2 \cos \phi_2$

Component of resultant current along Y – axis

= algebraic sum of I_1 and I_2 along Y – axis

or $I \sin \phi = I_1 \sin \phi_1 - I_2 \sin \phi_2$

$$\therefore \quad I = \sqrt{(I \cos\emptyset)^2 + (I \sin\emptyset)^2}$$

$$= \sqrt{(I_1 \cos\emptyset_1 + I_2 \cos\emptyset_2)^2 + (I_1 \sin\emptyset_1 - I_2 \sin\emptyset_2)^2}$$

and $\tan \phi = \frac{I_1 \sin\emptyset_1 - I_2 \sin\emptyset_2}{I_1 \cos\emptyset_1 + I_2 \cos\emptyset_2}$

If $\tan \phi$ is positive, current leads and if $\tan \phi$ is negative, then the current lags behind applied voltage V. power factor for the entire circuit

$\text{Cos}\phi = \frac{I_1 \cos\emptyset_1 + I_2 \cos\emptyset_2}{I}$

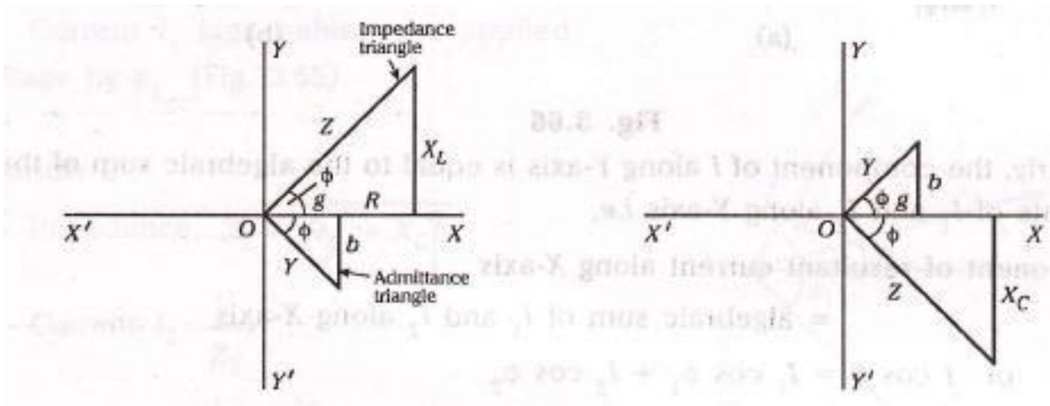
Admittance Method

The reciprocal of impedance of a circuit is called its admittance. It is represented by Y.

$$Y = \frac{1}{Z} = \frac{1}{V}$$

$$\text{So, } Y = \frac{\text{r.m.s.amperes}}{\text{r.m.s volts}}$$

Its unit is Siemens (S). A circuit with an impedance of one ohm has an admittance of one siemen. Earlier, the unit of admittance was mho.



(a) Inductive Circuit (b) Capacitive circuit

Fig. 3.67

Just as impedance Z of a circuit had two rectangular components, resistance R and reactance X, admittance Y also has two rectangular components known as conductance **g** and susceptance **b**. fig. 3.67 shows the impendence triangle and the admittance triangle. It is clear the admittance has two components **g** and **b.** The component **g** along the X- axis is the conductance which is the reciprocal of resistance. The component **b** is called susceptance, which is the reciprocal of reactance.

In fig. 3.67(a), the impedance and admittance triangles for an inductive circuit are shown. It is apparent that susceptance **b** is negative, being below X – axis. Hence inductive susceptance is negative. In fig. 3.67 (b), the impedance and admittance triangles for capacitive circuit is shown. It is evident that susceptance is positive, being above the X – axis; hence, capacitive susceptance is positive.

Relations

Conductance **g** = Y cos ϕ

Or $$g = \frac{1}{Z} \cdot \frac{R}{Z} = \frac{R}{Z^2} = \frac{R}{R^2 + X^2}$$

Conductance is always positive.

Susceptance **b** = Y sin $\phi = \frac{1}{Z} \cdot \frac{X}{Z} = \frac{X}{Z^2} = \frac{X}{R^2+X^2}$

Susceptance **b** is positive if reactance **X** is capacitive and negative if reactance is inductive.

Admittance $Y = \sqrt{g^2 + b^2}$

The units of g, b and y are in Siemens.

Application of admittance method

Let us consider a parallel circuit with three branches, as given in fig. 3.68.

we can determine the conductors by just adding the conductance of the three branches. In a like manner, susceptance is determined by the algebraic addition of the susceptances of the different branches.

Total conductance, $G = g_1 + g_2 + g_3$ Total susceptance

$B = (-b_1) + (-b_2) + b_3$

$\therefore$ Total admittance $Y = \sqrt{G^2 + B^2}$

Total current $I = VY$

Power factor, $\cos\phi = \frac{G}{Y}$

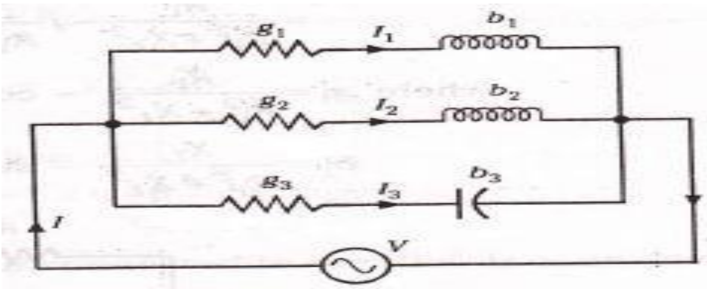


Fig. 3.68

Symbolic or j- method

Let us take the parallel two – branch circuit of fig. 3.69, with the same p.d. across the two impedances Z_1 and Z_2.

$I_1 = \frac{V}{Z_1}$ and $I_2 = \frac{V}{Z_2}$

Total current $I = I_1 + I_2 = \frac{V}{Z_1} + \frac{V}{Z_2} = V\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)$

$= V(Y_1 + Y_2)$

$= VY$

Where the total admittance $Y = Y_1 + Y_2$

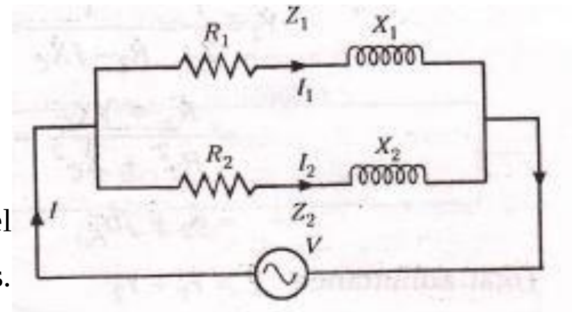


Fig. 3.69

We should note that admittances are added for parallel branches, whereas impedances are added for series branches. Both admittances and impedances are complex quantities, so all additions have to be performed in complex form.

In case of the two parallel branches of fig. 3.70,

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_L} = \frac{(R_1 - jX_L)}{(R_1 + jX_L)(R_1 - jX_L)} = \frac{R_1 - jX_L}{R_1^2 + X_L^2}$$

$$= \frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2} = g_1 - jb_1$$

Where $g_1 = \frac{R_1}{R_1^2 + X_L^2}$ ---- conductance of top branch

$b_1 = \frac{X_L}{R_1^2 + X_L^2}$ ---- susceptance of top branch

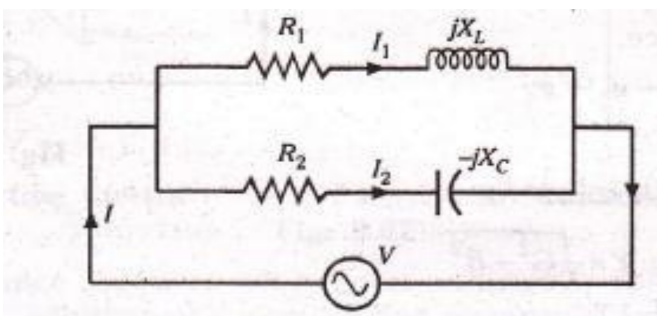


Fig. 3.70

In similar manner,

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 - jX_L} = \frac{(R_2 + jX_L)}{(R_2 - jX_C)(R_2 + jX_C)} = \frac{R_2 + jX_C}{R_2^2 + X_C^2}$$

$$= \frac{R_2}{R_2^2 + X_C^2} + j\frac{X_L}{R_2^2 + X_C^2} = g_2 + jb_2$$

Total admittance $Y = Y_1 + Y_2$

$$= (g_1 - jb_1) + (g_2 + jb_2)$$

$$= (g_1 + g_2) - j(b_1 - b_2)$$

$$= G - JB$$

$$Y = \sqrt{(g_1 + g_2)^2 + (b_1 - b_2)^2}$$

$$\phi = \tan^{-1}\left[\frac{b_1 - b_2}{g_1 + g_2}\right]$$

In polar form, admittance $Y = Y \angle \phi^0$

$$Y = \sqrt{G^2 + B^2} \angle -\tan^{-1}\left(\frac{B}{G}\right)$$

Total current $I = VY$; $I_1 = VY_1$ and I_2 VY_2

$V = V \angle 0^0$ and $Y = Y \angle \phi$

So $I = VY = V \angle 0^0 \times Y \angle \phi = VY \angle \phi$

Taking a general case,

$V = V \angle \propto$ and $Y = Y \angle \beta$, then

So $I = VY = V \angle \propto \times Y \angle \beta = VY \angle \propto + \beta$