

MODEL QUESTION PAPER SET-1 SOLUTIONS
2022-23

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- 1 a. State and explain Heisenberg's uncertainty principle. Show that an electron does not exist inside the nucleus on the basis of HUP.

Statement of Heisenberg's uncertainty principle:

"The HUP states that
"It is impossible to determine both the exact position and the exact momentum of a particle at the same time. The product of uncertainties in these quantities is always greater than or equal to $(h/4\pi)$ ".

ie $\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$

$\Delta x \rightarrow$ is uncertainty in position
 $\Delta p \rightarrow$ is " in momentum.

III $\Delta E \cdot \Delta t \geq \frac{h}{4\pi}$ represents in energy & time

$\Delta L \cdot \Delta \theta \geq \frac{h}{4\pi}$ represents in angular momentum (ΔL) and angular displacement ($\Delta \theta$).

Show that the electrons cannot exist inside the nucleus using HUP.

Electron to be present in the nucleus, the maximum uncertainty in position

$$\Delta x = 10^{-14} \text{ m (diameter)}$$

According to HUP

The minimum uncertainty in momentum

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\Delta p \geq \frac{h}{4\pi \cdot \Delta x}$$

$$\Delta p \geq \frac{6.63 \times 10^{-34}}{4\pi \times 10^{-14}}$$

$$\Delta p \geq 5.275 \times 10^{-21} \text{ kg m/s} = p \text{ (say).}$$

The minimum energy of the electron in the nucleus is given by

$$\therefore E = \frac{p^2}{2m}$$

$$\geq \frac{(5.275 \times 10^{-21})^2}{2 \times 9.1 \times 10^{-31}}$$

$$\geq 1.527 \times 10^{-11} \text{ J}$$

$$\geq \frac{1.527 \times 10^{-11}}{1.6 \times 10^{-19}} \text{ eV} = 95.45 \text{ MeV} = 95 \text{ MeV}$$

But the maximum K.E. of the $\bar{e}$ (β -particle) emitted from the nucleus does not exceed 4 MeV. Hence electrons do not present in the nucleus.

1b. What is wave function? Give its physical significance & properties.

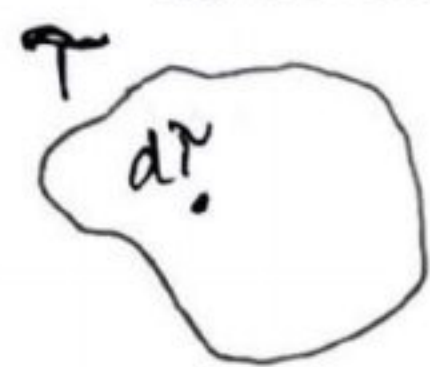
Ans: Wave function:

The quantity whose periodic variations make up matter waves is called wave function.

Every wave is characterised by periodic variations in some physical quantity.

Physical Significance:

1. Probability density:



If ψ is the wave function associated with a particle, then $|\psi|^2$ is the probability of finding a particle in unit volume. If "V" is the volume in which the particle is present but where it is exactly present is not known. Then the probability density of finding a particle in certain elemental volume dV is given by

$$|\psi|^2 dV.$$

Thus $|\psi|^2$ is the probability density. The p.d. of finding an event is real & positive quantity.

In case of complex wave functions.

$$|\psi|^2 = \psi \times \psi^*$$

where ψ^* is complex conjugate of ψ .

Normalization:

The probability of finding a particle having wave function " ψ " in a volume " $d\tau$ " is $|\psi|^2 d\tau$. If it is certain that the particle is present in finite volume " τ " then

$$\int_0^{\tau} |\psi|^2 d\tau = 1$$

$$\int_{-\infty}^{+\infty} |\psi|^2 d\tau = 1$$

Properties of wave functions:

1. Wave function (ψ) is single valued everywhere
2. ψ is finite everywhere
3. ψ is continuous everywhere
4. ψ & its first ^{derivatives w.r.t.} variables. ~~not~~

1 c. An electron is bound in a 1-dimensional potential well of width $1\text{\AA}$ & of infinite height. Find its energy values in eV in the ground state & also in the first two excited state

Soln.

Given data:

width of potential well $a = 1\text{\AA} = 10^{-10}\text{ m}$

Equation of Energy Eigen values is

$$E_n = \frac{n^2 h^2}{8ma^2}$$

Planck's constant

$$h = 6.63 \times 10^{-34} \text{ J-s}$$

mass of electron

$$e = 9.1 \times 10^{-31} \text{ kg}$$

$$n=1$$

$$E_1 = \frac{h^2}{8ma^2}$$

$$= \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2}$$

$$= 0.6038 \times 10^{-18} \text{ J}$$

$$E_1 = 6.038 \times 10^{-19} \text{ J}$$

$$E_1 = \frac{6.038 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 3.7737 \times 10 \text{ eV}$$

$$E_1 = 37.737 \text{ eV}$$

Energy of ground state

Energy of first excited state ($n=2$)

$$E_2 = 4E_1$$

$$= 4 \times 37.737 =$$

$$E_2 = 150.948 \text{ eV}$$

Energy of second excited state ($n=3$)

$$E_3 = 9E_1$$

$$E_3 = 339.633 \text{ eV}$$

2 a. Setup time independent Schrodinger's wave equation for free particle in one-dimension.

Consider a particle of mass 'm' moving with velocity 'v'.

The de-Broglie wavelength ' λ ' is

$$\lambda = \frac{h}{mv} = \frac{h}{p} \quad \text{--- (1)}$$

where 'mv' is the momentum of the particle.

The wave eqn is 1-dimensional wave function ψ for the de-Broglie wave of a particle moving along the direction of x-axis is given by $i(kx - \omega t)$

$$\psi = A e^{i(kx - \omega t)} \quad \text{--- (2)}$$

where A is a constant and

$\omega \rightarrow$ is the angular frequency of the wave. where $A e^{ikx}$ represent the time independent part of the wave function & is represented by $\psi = A e^{ikx}$

Differentiating eqn (2) w.r.t 't' twice, we get

$$\frac{d\psi}{dt} = (-i\omega) e^{i(kx - \omega t)} \quad \&$$

$$\frac{d^2\psi}{dt^2} = (-i\omega)^2 e^{i(kx - \omega t)} = (-1)\omega^2 A e^{i(kx - \omega t)}$$

$$\frac{d^2\psi}{dt^2} = -\omega^2 \psi \quad \text{--- (3)}$$

The equation of a travelling wave is

$$\frac{d^2y}{dx^2} = \frac{1}{v^2} \frac{d^2\psi}{dt^2} \quad \text{--- (4)}$$

where y is displacement &
v is the velocity.

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Similarly, for the de-Broglie wave associated with the particle.

By analogy, we can write the wave equation for the de-Broglie wave for the motion of a free particle as

Here, $y = \psi$

$$\frac{d^2\psi}{dx^2} = \frac{1}{v^2} \frac{d^2\psi}{dt^2} \quad \text{--- (4)}$$

The above eqn represents wave propagating along the x -axis with a velocity ' v ' & ψ is the displacement at time ' t '.

From eqn (3) & (4), we get

$$\frac{d^2\psi}{dx^2} = -\frac{\omega^2}{v^2} \psi \quad \text{--- (5)}$$

If λ and ν are the wavelength & frequency of the wave, then

But $\omega = 2\pi f = 2\pi\nu$ | w.k.t $c = \nu\lambda$ $\nu = \frac{c}{\lambda}$

ν be the frequency of radiation &
 λ be the wavelength of the radiations

The angular frequency,

$$\omega = 2\pi\nu = \frac{2\pi c}{\lambda}$$

Squaring on both sides, we get

$$\omega^2 = 4\pi^2 \frac{c^2}{\lambda^2} \quad \text{or} \quad v^2 = \frac{\omega^2 \lambda^2}{4\pi^2}$$

Put the value of v^2 in eqn (5), we get

$$\frac{d^2\psi}{dx^2} = -\frac{4\pi^2}{\lambda^2} \psi \quad \text{--- (6)}$$

$$\frac{1}{\lambda^2} = -\frac{1}{4\pi^2\psi} \cdot \frac{d^2\psi}{dx^2} \quad \text{--- (7)}$$

For a particle of mass 'm' moving with a velocity 'v'

$$\text{Kinetic energy} = K.E = \frac{1}{2}mv^2$$

$$K.E = \frac{m^2v^2}{2m} \quad (\times \& \div \text{ by "m"})$$

$$K.E = \frac{p^2}{2m} \quad \text{--- (8)} \quad (p = mv)$$

But from equⁿ (1), we have

But de-Broglie wavelength $\lambda = \frac{h}{p}$

$$p = \frac{h}{\lambda}$$

By substituting for 'p' in equⁿ (8)

$$K.E = \frac{(h/\lambda)^2}{2m}$$

$$K.E = \frac{h^2}{2m\lambda^2}$$

$$\therefore K.E = \frac{h^2}{2m} \cdot \frac{1}{\lambda^2} \quad \text{--- (9)}$$

By substituting $\frac{1}{\lambda^2}$ from the equⁿ (7)

Equⁿ (9) becomes

$$K.E = \frac{h^2}{2m} \left[-\frac{1}{4\pi^2\psi} \frac{d^2\psi}{dx^2} \right]$$

$$K.E = -\frac{h^2}{8\pi^2m\psi} \cdot \frac{d^2\psi}{dx^2} \quad \text{--- (10)}$$

Let there be a field where the particle is present. Depending on its position in the field, the particle will access certain potential energy 'V'. Then, the total energy E of the particle is the sum of K.E & the potential energy (V) is given by

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Total energy (E) = Kinetic energy + Potential energy

$$E = -\frac{h^2}{8\pi^2m} \cdot \frac{1}{\psi} \cdot \frac{d^2\psi}{dx^2} + V$$

$$-\frac{h^2}{8\pi^2m} \cdot \frac{1}{\psi} \cdot \frac{d^2\psi}{dx^2} = E - V$$

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2}(E - V)\psi = 0} \quad \text{--- (1)}$$

This is Schrodinger's wave equation for a particle.

2 b.

Discuss the wave functions, probability densities & energy level for a particle in a box by considering the ground & the first two excited state.

Let us consider the most probable location of the particle in the well & its energies for first three cases.

Case (i): $n=1$

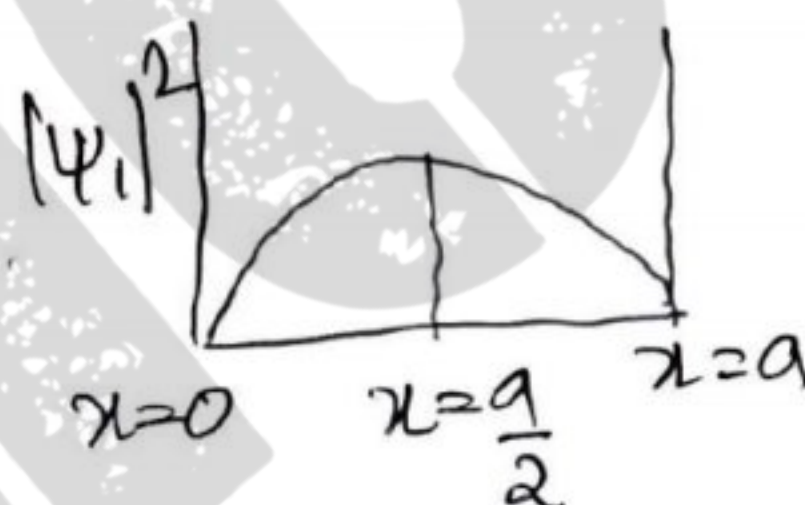
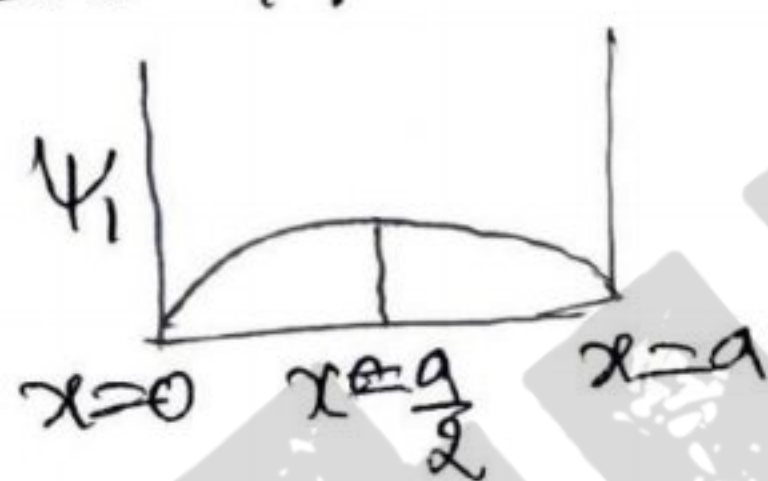
It is the ground & the particle is normally present in this state

The Eigen function is

$$\psi_1 = \sqrt{\frac{2}{a}} \cdot \sin\left(\frac{\pi}{a}\right)x$$

$$\psi_1 = 0 \text{ for } x=0 \text{ \& } x=a$$

But ψ_1 is maximum when $x = \frac{a}{2}$



The plots of ψ_1 versus x & $|\psi_1|^2$ versus x are shown in the above figure.

$|\psi_1|^2 = 0$ for $x=0$ & $x=a$ & it is max. for $x = \frac{a}{2}$
i.e. in ground state the particle cannot be found at the walls, but the probability of finding it is maximum in the middle.
The energy of the particle at the ground state is

$$E_1 = \frac{n^2 h^2}{8ma^2} = E_0$$

Case: (ii) $n=2$

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In the first excited state the Eigen function of this state is

$$\psi_2 = \sqrt{\frac{2}{a}} \cdot \sin\left(\frac{2\pi}{a}\right)x$$

$\psi_2 = 0$ for the values $x=0, a/2, a$.

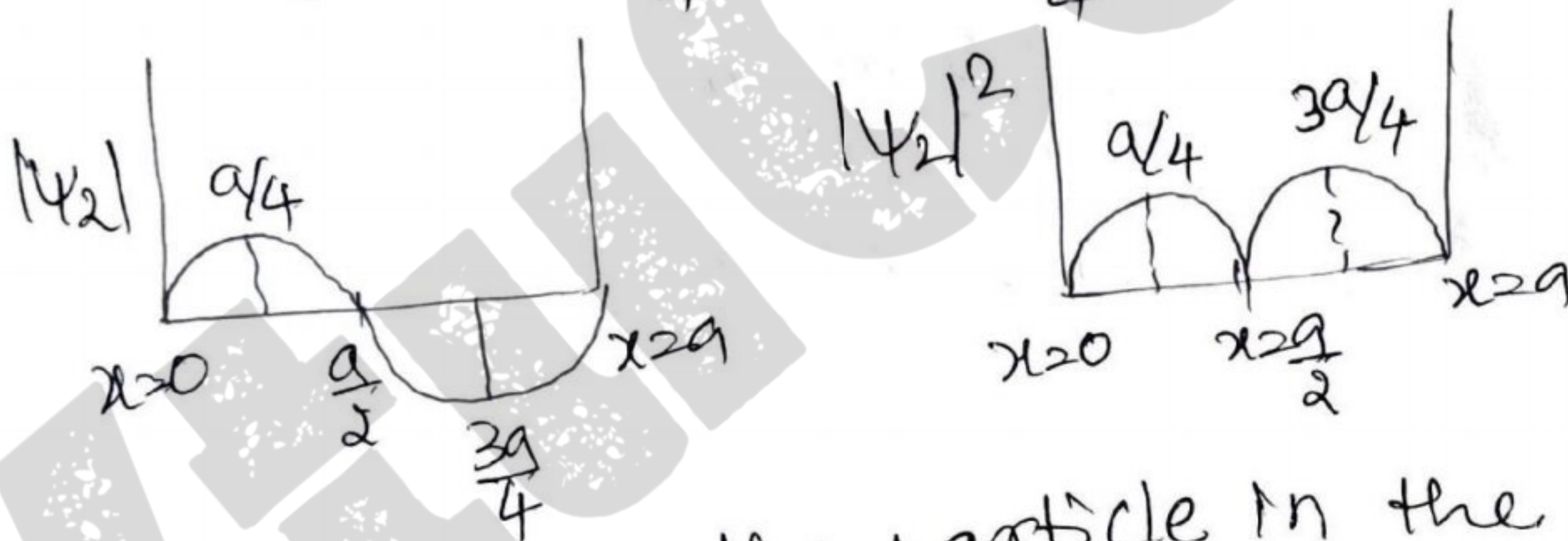
Also ψ_2 is maximum for the values $x=a/4$ & $\frac{3a}{4}$

These are represented in the graphs.

$$|\psi_1|^2 = 0 \text{ at } x=0, a/2, a$$

i.e. Particle cannot be found either at the walls or at the centre $|\psi_2|^2 = \text{maximum}$

for $x = \frac{a}{4}$ & $x = \frac{3a}{4}$



The energy of the particle in the first excited state is E_2

$$E_2 = 4E_0$$

2C. Calculate the de-Broglie wavelength associated with an electron having a kinetic energy of 100 eV

Given data:

$$\lambda = ?$$

~~mass~~ $K.E = 100 \text{ eV}$

de-Broglie wavelength

$$\lambda = \frac{12.26 \times 10^{-10} \text{ m}}{\sqrt{V}}$$

$$= \frac{12.26 \times 10^{-10} \text{ m}}{\sqrt{100}}$$

$$\lambda = \frac{12.26 \times 10^{-10} \text{ m}}{10}$$

$$\lambda = 12.26 \times 10^{-11} \text{ m}$$

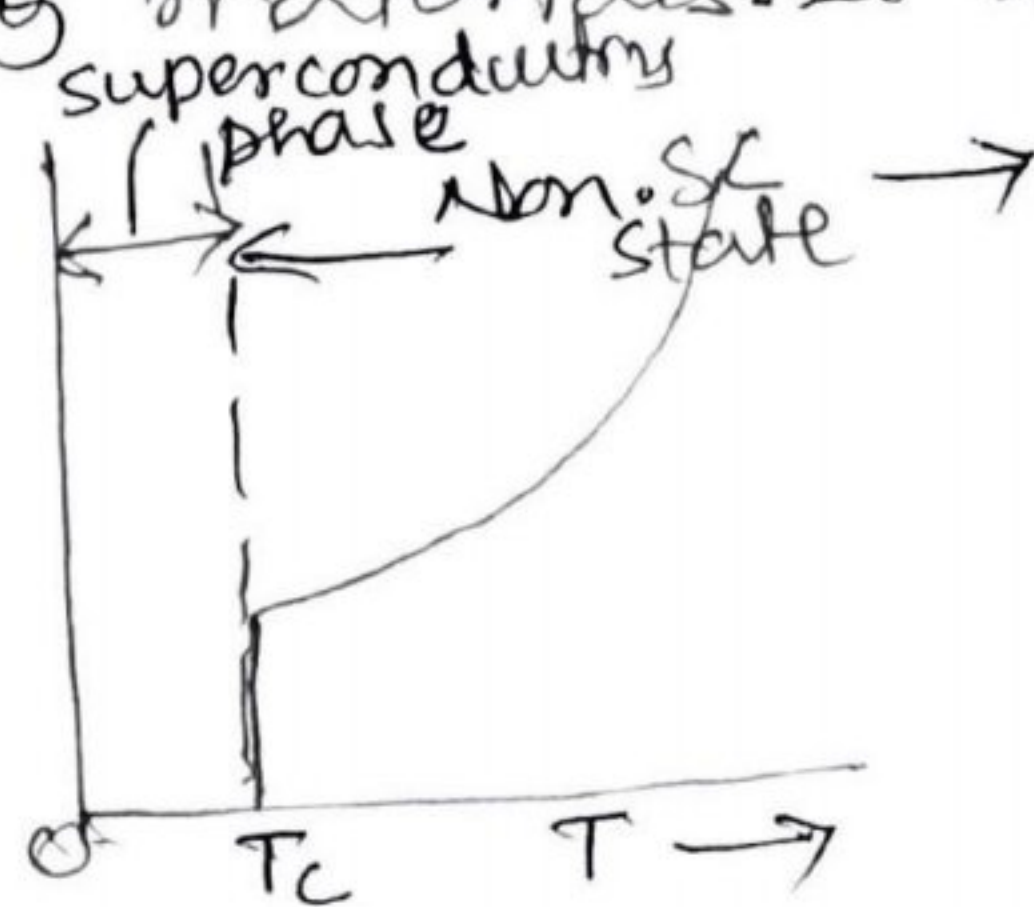
2. a. Explain the dependence of resistance on temperature of a semiconductor.
Describe Type I & type II superconductors.

Temperature dependence of resistivity of a superconductor:

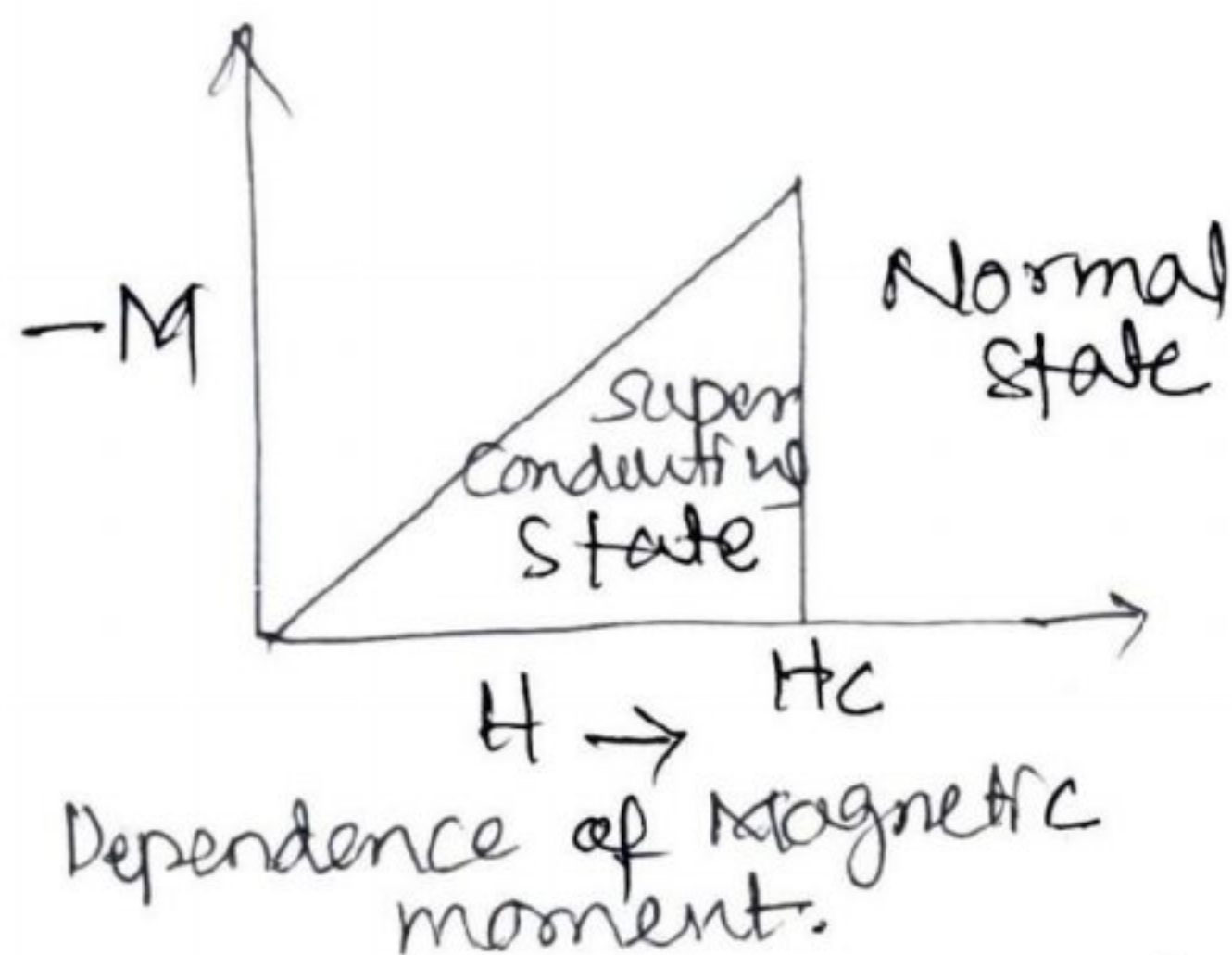
The resistance of a superconductor in the non-superconducting state decreases with temperature & the electrical conductivity of some of the metals & alloys vanish entirely below a certain temperature.

"The resistance offered by certain materials to the flow of electric current abruptly drop to zero below or threshold temperature. This phenomenon is called superconductivity & threshold temperature is called "critical temperature." "The temperature at which a material undergoes transition from normal state to superconducting state losing its resistivity is called the critical temperature or transition temperature T_c ".

Above the transition temp, the substance is in the normal state & below it will be in superconducting state. The critical temp is different for the different superconducting materials. It is not sensitive to the presence of small amount of impurities



Type I superconductors:



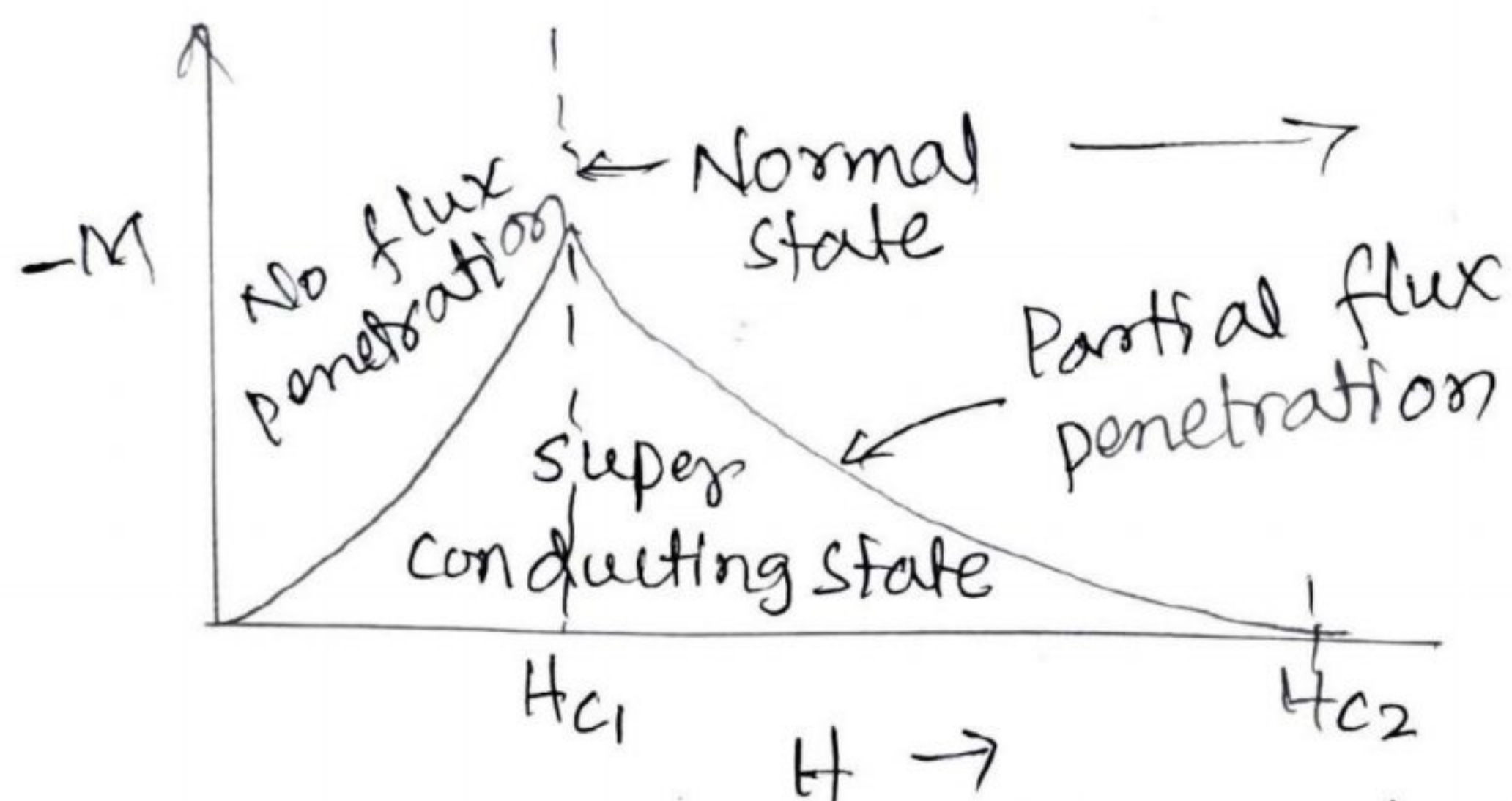
These materials exhibit complete Meissner's effect & have well defined critical field H_c . These are perfect diamagnetic in the superconducting state and possesses -ve magnetic moment.

The material remains in the superconducting when the field is less than the critical field. It repels the magnetic field lines of force from the body of the matter immediately after H_c , the material transitions to normal state & the flux density penetrates the material. i.e. Meissner's effect is absent. H_c is of the order of 0.1 T or less.

Since H_c is very low, even weak magnetic field can destroy the phenomenon.

As weak magnetic field can penetrate the material more easily & they are also called soft superconductors.

Type-II superconductor:



These materials are having two critical fields H_{c1} & H_{c2} . For the fields less than H_{c1} ($H < H_{c1}$) (lower critical field), it expels the magnetic field completely & there is no flux penetration. It becomes a perfect diamagnetic & the material is in the superconducting state. After H_{c1} , the flux penetrates & partially fills the body of the material through channels called filaments. As the field is increased these filaments broaden & by H_{c2} [the material possesses both normal & superconducting state] Hence the state is called Mixed state.

Type-II SC's can carry larger currents when the magnetic field is between H_{c1} & H_{c2} . H_{c2} → upper critical field is many a folds greater than H_{c1} , the lower critical fields.

Only strong magnetic field of the order of 10 T can penetrate the material, hence these are called Hard superconductors. Used in SC's magnets of high magnetic fields above 10 T.

3b. Derive classical Mossotti equation.

Consider an elemental solid dielectric material of dielectric constant ϵ_r .

If N is the number of atoms/unit volume of the material.

It is the atomic dipole moment, then we have,

$$\text{Dipole moment/unit volume} = N\mu \quad \text{--- (1)}$$

Here the field experienced by the atoms in the internal field E_i .

Hence, if α_e is the electronic polarizability of the atoms, we can write the equation for μ as,

$$\mu = \alpha_e E_i \quad \text{--- (2)}$$

$\therefore$ Equⁿ (1) becomes,

$$\text{dipole moment/unit volume} = N\alpha_e E_i \quad \text{--- (3)}$$

In equⁿ (3), its left side is same as polarization P ,

$$P = N\alpha_e E_i \quad \text{--- (4)}$$

$$\text{or} \quad E_i = \frac{P}{N\alpha_e} \quad \text{--- (5)}$$

But we have the relation for P as,

$$P = \epsilon_0 (\epsilon_r - 1) E$$

where E is the applied field.

$$E = \frac{P}{\epsilon_0 (\epsilon_r - 1)} \quad \text{--- (6)}$$

Also, we have the equation for internal field as,

$$E_i = E + \gamma \frac{P}{\epsilon_0} \quad \text{--- (7)}$$

where, γ is the internal field constant.

Substituting for E_i and E from eqn (5) & (6) in eqn (7), we have

$$\frac{P}{N \alpha_e} = \frac{P}{\epsilon_0 (\epsilon_r - 1)} + \gamma \frac{P}{\epsilon_0}$$

$$\text{or } \frac{1}{N \alpha_e} = \frac{1}{\epsilon_0} \left[\frac{1}{(\epsilon_r - 1)} + \gamma \right]$$

Considering the internal field in the material to be Lorentz field, we have $\boxed{\gamma = \frac{1}{3}}$

Substituting the same in the above equation, we get

$$\begin{aligned} \frac{1}{N \alpha_e} &= \frac{1}{\epsilon_0} \left[\frac{1}{(\epsilon_r - 1)} + \frac{1}{3} \right] \\ &= \frac{1}{\epsilon_0} \left[\frac{3 + \epsilon_r - 1}{3(\epsilon_r - 1)} \right] \end{aligned}$$

$$\frac{1}{N\alpha_e} = \frac{1}{\epsilon_0} \left[\frac{3 + \epsilon_r - 1}{3(\epsilon_r - 1)} \right]$$

$$\frac{\epsilon_0}{N\alpha_e} = \frac{\epsilon_r + 2}{3(\epsilon_r - 1)}$$

By rearranging the above equation

$$\boxed{\frac{(\epsilon_r - 1)}{(\epsilon_r + 2)} = \frac{N\alpha_e}{3\epsilon_0}} \quad \text{--- (8)}$$

Eqn (8) is called Clausius-Mossotti equation & it holds good for crystals of high degree of symmetry. The equation provides a relation between ϵ_r , the dielectric constant which is a microscopically measurable quantity, & α_e is electronic polarizability which is an atomic property i.e. microscopic.

3c. Show that the sum of the probability of occupation ^{of an energy} state at ΔE below Fermi level & that at ΔE above Fermi level is unity. 19

To prove:

$$[\text{occupation probability at } E = (E_F + \Delta E)] + [\text{occupation probability at } E = (E_F - \Delta E)] = 1$$

Proof:

The occupation probability for an energy state E at a temperature T is given by the fermi factor,

$$f(E) = \frac{1}{e^{\frac{E-E_F}{KT}} + 1} \quad \text{--- (1)}$$

For an energy state at ΔE above Fermi level, the energy is $E = (E_F + \Delta E)$

By substituting in equⁿ (1), we get

$$[\text{Occupation probability}]_{E=E_F+\Delta E} = \frac{1}{e^{\frac{E-E_F}{KT}} + 1} \quad \text{--- (2)}$$

Similarly for the energy state at ΔE below Fermi level,

$E = (E_F - \Delta E)$, we get

$$[\text{Occupation probability}]_{E=(E_F-\Delta E)} = \frac{1}{e^{\frac{E-E_F}{KT}} + 1}$$

Let us put $e^{\frac{\Delta E}{kT}} = x$, and

add eqn (2) and eqn (3),

Then, the sum of the probability of occupancy of the energy states at ΔE below & above E_F

$$\begin{aligned} E_F &= \frac{1}{(x+1)} + \frac{1}{(\frac{1}{x}+1)} \\ &= \frac{1}{(x+1)} + \frac{x}{(x+1)} \\ &= \frac{1+x}{1+x} = 1 \end{aligned}$$

$\therefore$ Sum of the probabilities = 1

4a. Mention any three assumptions of Quantum free electron theory? Discuss the probability of occupation of energy levels.

Assumptions of QFET:

1. The energy values of the conduction electrons are quantized. The allowed energy values are realized in terms of a set of energy levels.
2. The distribution of electrons in the various allowed energy levels occurs as per Pauli's exclusion principle.
3. The electrons cannot escape from the nucleus due to potential barrier inside the atoms.
4. The force of attraction between conduction free electrons with lattice ions and force of repulsion between themselves are negligible.

Dependence of Fermi factor on temperature & Effects on occupancy of Energy levels:

The dependence of Fermi factor on temp. & the effect on occupancy of energy levels is as shown in figure.

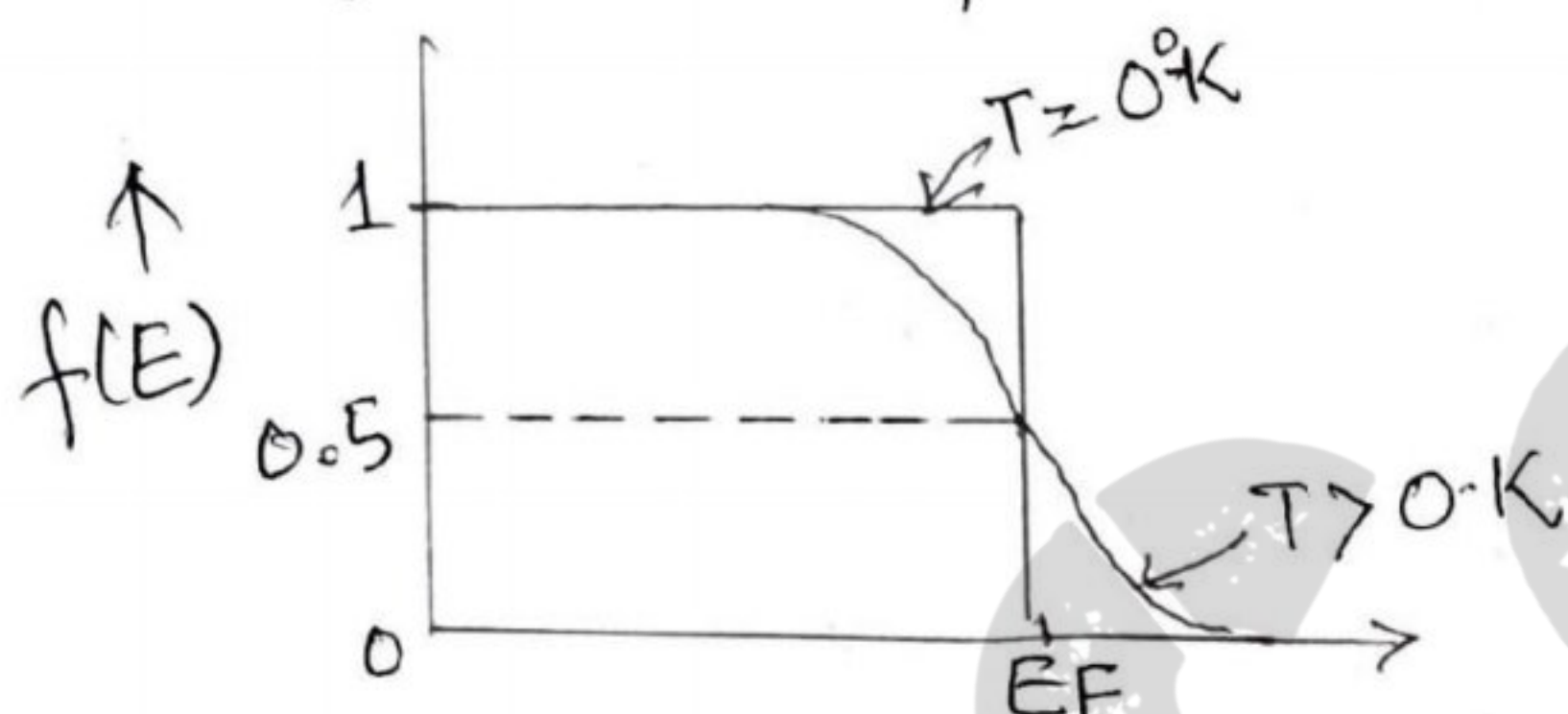
Let us consider the different cases of distribution as follows

(i) Probability of occupation for $E < E_F$ at $T = 0^\circ K$

When $T = 0$ and $E < E_F$, we have for the probability.

$$f(E) = \frac{1}{e^{\frac{E - E_F}{kT}} + 1} = \frac{1}{e^{-\infty} + 1} = \frac{1}{0 + 1}$$

$$f(E) = 1 \text{ for } E < E_F$$



Variation of $f(E)$ with E .

Here $f(E) = 1$ means the energy level is certainly occupied & $E < E_F$ applied to all the energy levels below E_F .

At $T = 0$, all the energy levels below the Fermi level are occupied.

(ii) Probability of occupation for $E > E_F$ at $T = 0^\circ K$

When $T = 0$ & $E > E_F$

$$f(E) = \frac{1}{e^{\frac{E - E_F}{kT}} + 1} = \frac{1}{\infty + 1} = \frac{1}{\infty} = \frac{0}{1} = 0$$

$$f(E) = 0 \text{ for } E > E_F$$

$\therefore$ At $T = 0$, all the energy levels above Fermi level are unoccupied.

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In view of the above two cases, at $T = 0\text{ K}$ the variation of $f(E)$ for different energy values, becomes a step function as shown in fig (1).

(ii) Probability of occupation at ordinary temp.
($E = E_F$ at $T > 0^\circ\text{K}$.)

At ordinary temperatures, though the value of probability remains 1 for $E < E_F$ it starts decreasing from 1 as the values of E becomes to E_F .

The value of $f(E)$ becomes $\frac{1}{2}$ at $E = E_F$.

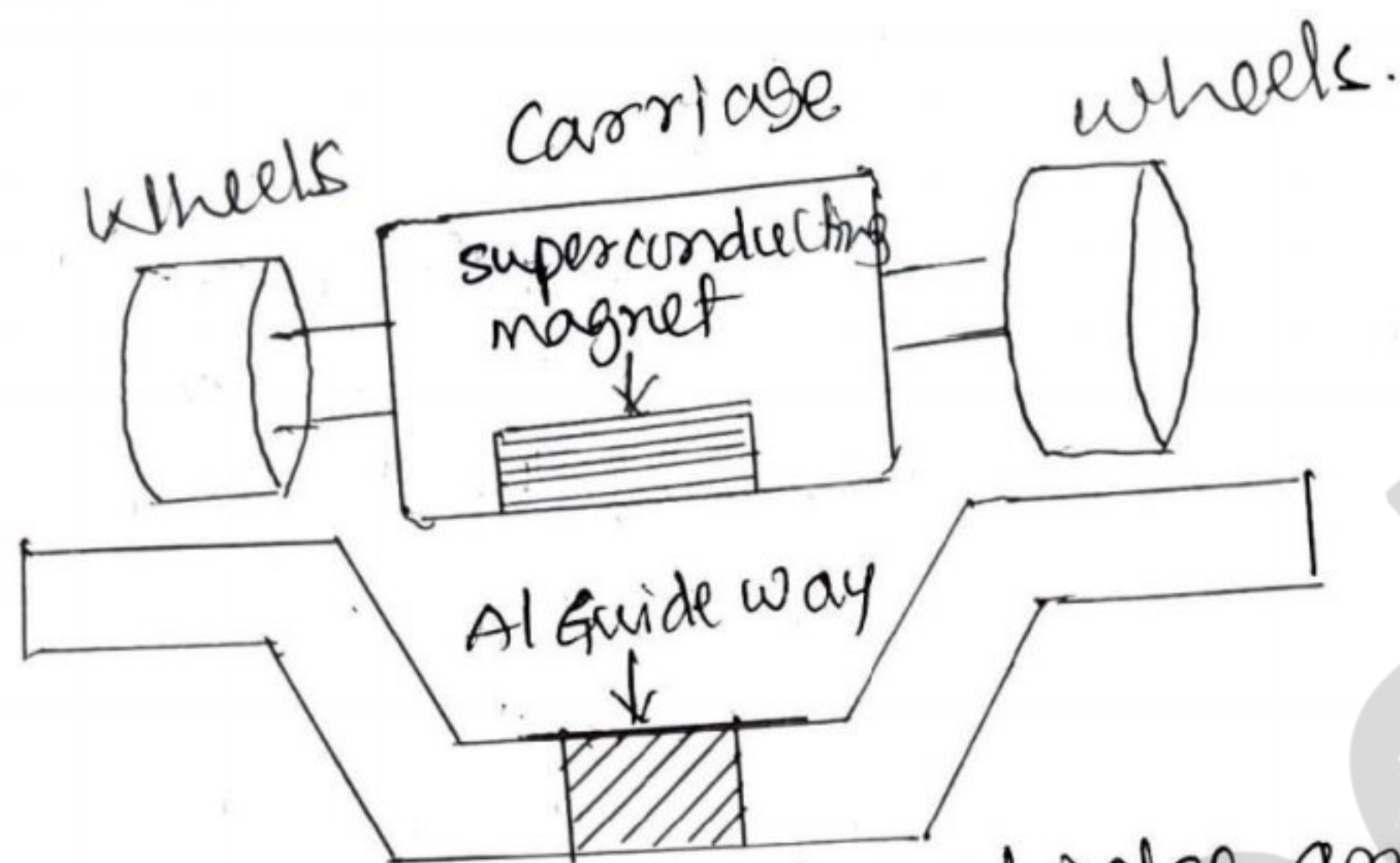
This is because, for $E = E_F$.

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1} = \frac{1}{e^0 + 1} = \frac{1}{1+1} = \frac{1}{2}$$

Further, for $E > E_F$, the probability value falls off to zero rapidly.

4 b. Explain the construction & working of MAGLEV vehicles.

Magnetically Levitated Vehicles (MAGLEV vehicles)



Magnetically levitated vehicles are called maglev vehicles. The ~~magnitude~~ magnetic levitation is based on the principle of Meissner's effect. The repulsion between the two strong fields produced by the superconducting magnet & electric current levitates the vehicles & is set a float. The floating of the vehicles by this principle is called Magnetic Levitation.

The superconducting magnet is kept inside the vehicle & the electric current flows in the Aluminium guide way. Once the magnetic field in the vehicle is switched on is repelled by the magnetic field of the guide way. As it is about to levitate, the guide way propels the vehicle forward of the guide way. As it is about to levitate,

the guide way propels the vehicle forward. by the segments provided in the Aluminium guide way. propels the vehicle forward by the segments provided in the Al guide way. The vehicle is provided with retractable wheels. Initially the vehicle runs on the guide way, once it is levitated in air the wheels are retracted into the body. The height to which the vehicle is levitated above guide way is about 10 cm to 15 cm. While stopping, the wheels are drawn out the the vehicle slowly settles on the guide way by running a distance. Since such vehicles float under magnetic effect, they do not have friction, less power consumption and noiseless travel.

4 c. The dielectric constant of helium gas at NTP is 1.0000684. Calculate the electronic polarizability of the atoms, if the helium gas contains 2.7×10^{25} atoms/m³.

Given data:

Dielectric constant of the gas at NTP,

$$\epsilon_r = 1.0000684,$$

Number density of He-atoms, $N = 2.7 \times 10^{25} \text{ /m}^3$

To find:

Electronic polarizability, $\alpha_e = ?$

Solution:

The electronic polarizability, α_e , for a rare gas atoms is given by

$$\alpha_e = 4\pi\epsilon_0 R^3$$

$$R = \left[\frac{\alpha_e}{4\pi\epsilon_0} \right]^{1/3} \quad \text{--- (1)}$$

$$\text{But, } \alpha_e = \frac{\epsilon_0(\epsilon_r - 1)}{N}$$

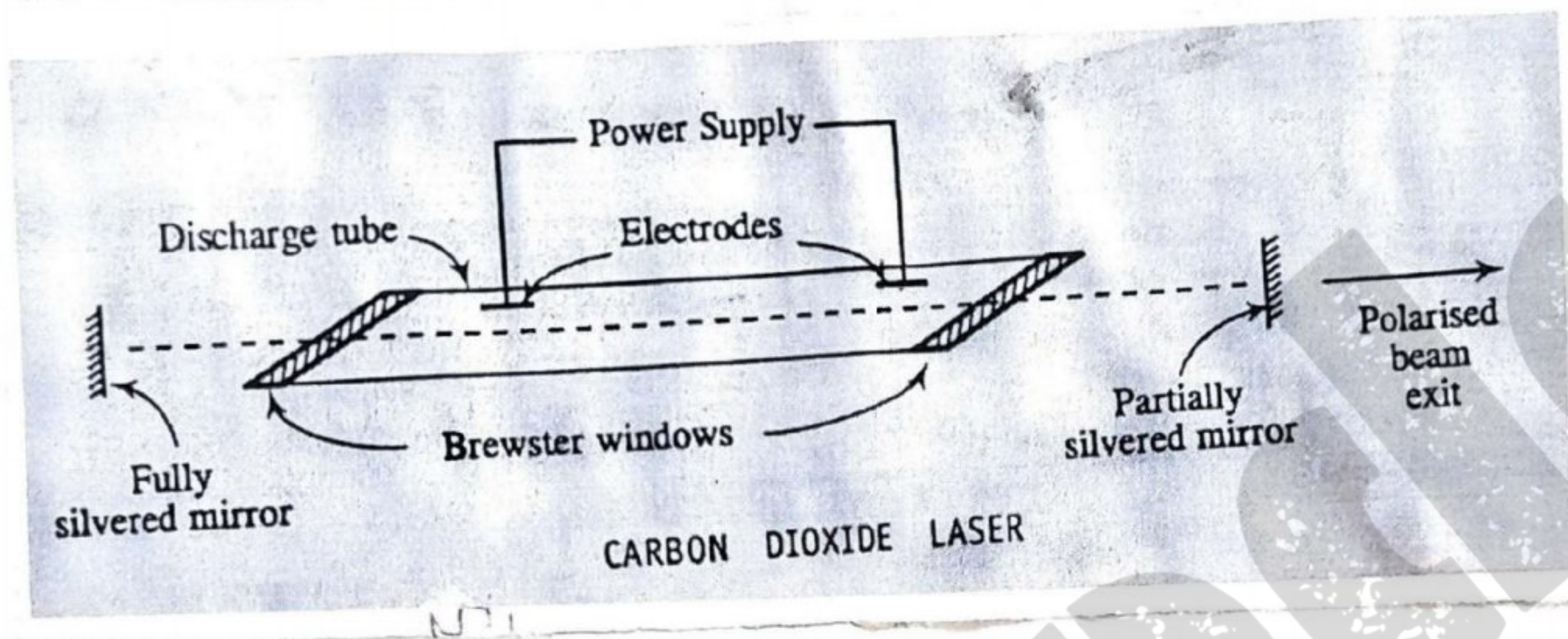
$$= \frac{8.83 \times 10^{-12} (1.0000684 - 1)}{2.7 \times 10^{25}}$$

$$\alpha_e = 2.242 \times 10^{-41} \text{ F-m}^2.$$

5a. Describe the principle, construction & working of Carbon Dioxide laser with energy level diagram.

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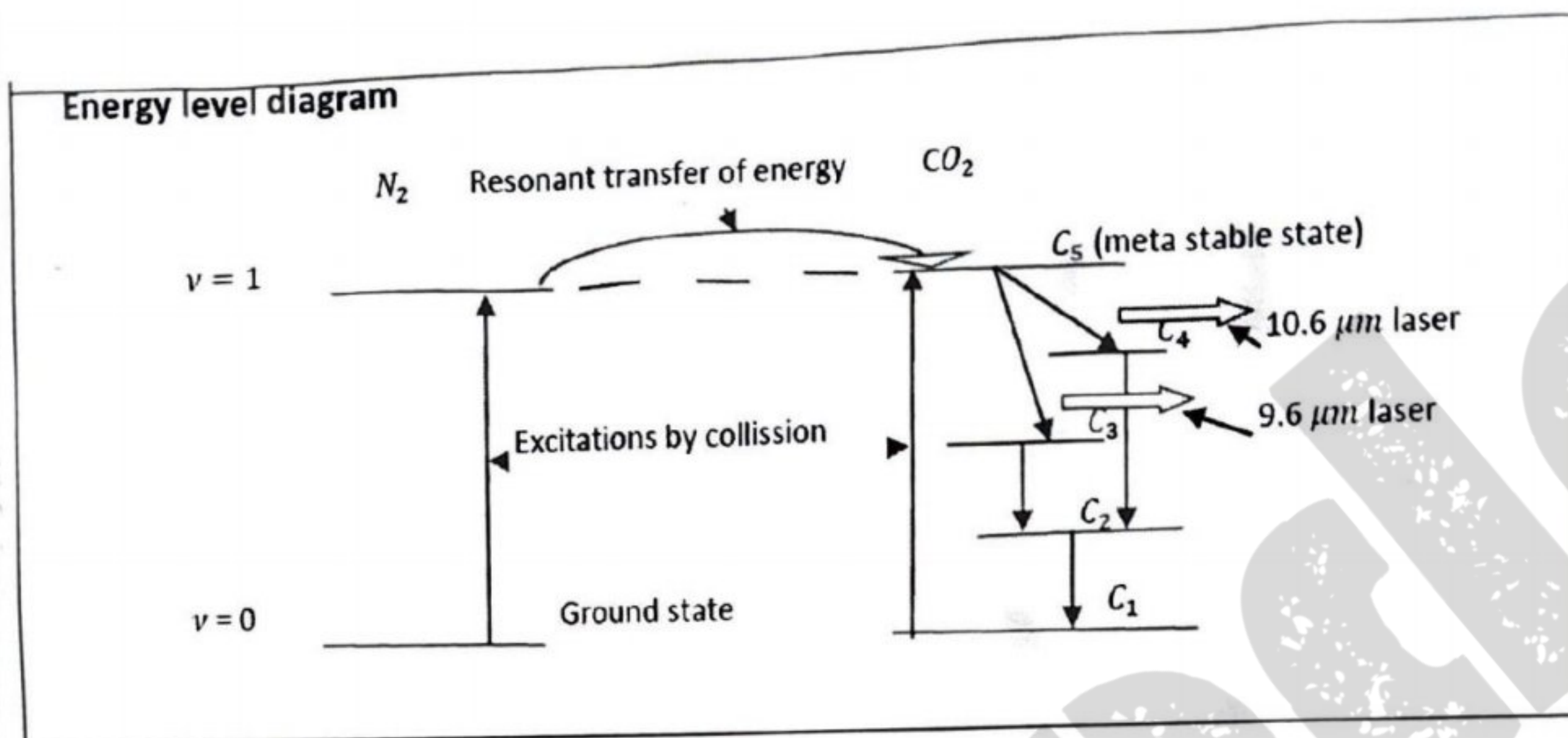
Construction:



It consists of glass quartz tube 5m long & 2.5cm in the diameter. This discharge tube is filled with gaseous mixture of CO_2 (active medium), He, Ni with suitable partial pressures. Water vapours are added as an additive. The terminals of the discharge tubes are connected to a D.C. power supply. The ends of the discharge tube are fitted with NaCl Brewster windows, so that the laser light generated will be polarized. Two concave mirrors one fully reflecting & the other partially form an optical resonator. The active centers are CO_2 molecules & lasing takes place on the transitions between the vibrational levels of the electronic ground state.

Working of CO₂ Laser:

Working level diagram:



1. When a suitable voltage is applied across the two electrodes, a glow discharge of the gases is initiated in the tube. During discharge many electrons are rendered free from the gas atoms; these free electrons begin colliding with the N₂ and O₂ molecules in their path. These accelerated electrons collide with N₂ molecules & excite them to higher energy levels. This process represented by the equation

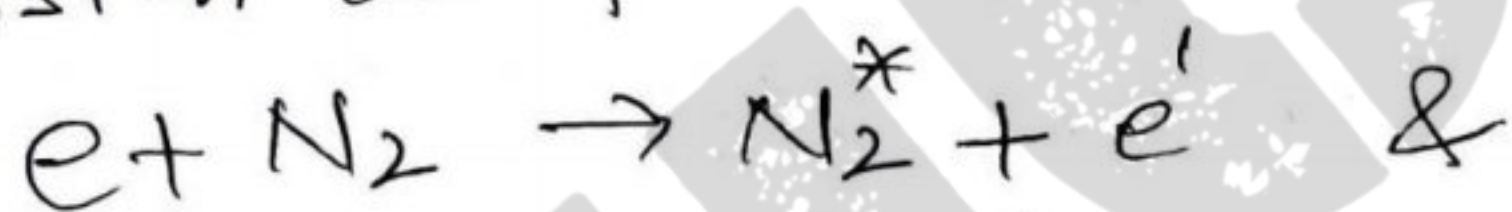
principle:

CO₂ laser works on the principle of stimulated emission.

Working:

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1. CO₂ laser is a four level molecular gas laser which produce continuous or pulsed laser beam.
2. It works on the principle of stimulated emission between the rotational sub levels of an upper & lower vibrational levels of CO₂ molecules.
3. Ionisation takes place due to electric discharge when high DC voltage is applied between electrodes producing electrons.
4. The accelerated electrons excite both N₂ & CO₂ atoms to their higher energy levels $v=1$ & C₅ from their ground states 0 & C₁ due to collision as follows:



where e & e' are the energies of electron before & after collision.

5. N₂^{*} molecules in excited level collide with CO₂ molecules in their ground state C₁ & excite it to metastable state C₅ by resonant energy transfer as level C₅ of CO₂ is same as level $v=1$ of N₂ given by



6. As this process continues due to electric discharge pumping, pumping inversion takes place between C_5 & C_4 and C_5 & C_3 .

7. The transitions / de-excitations takes place as follows:

$C_5 \rightarrow C_4$ producing laser $10.6 \mu\text{m}$ (IR region)

$C_5 \rightarrow C_3$ " " " $9.6 \mu\text{m}$ (")

$C_4 \rightarrow C_2$

$C_3 \rightarrow C_2$ Radiation less transition

$C_2 \rightarrow C_1$

8. Due to high thermal conductivity of He , it removes ~~heat~~ ^{heat} from mixture & de-populate the lower states C_3 & C_2 quickly.

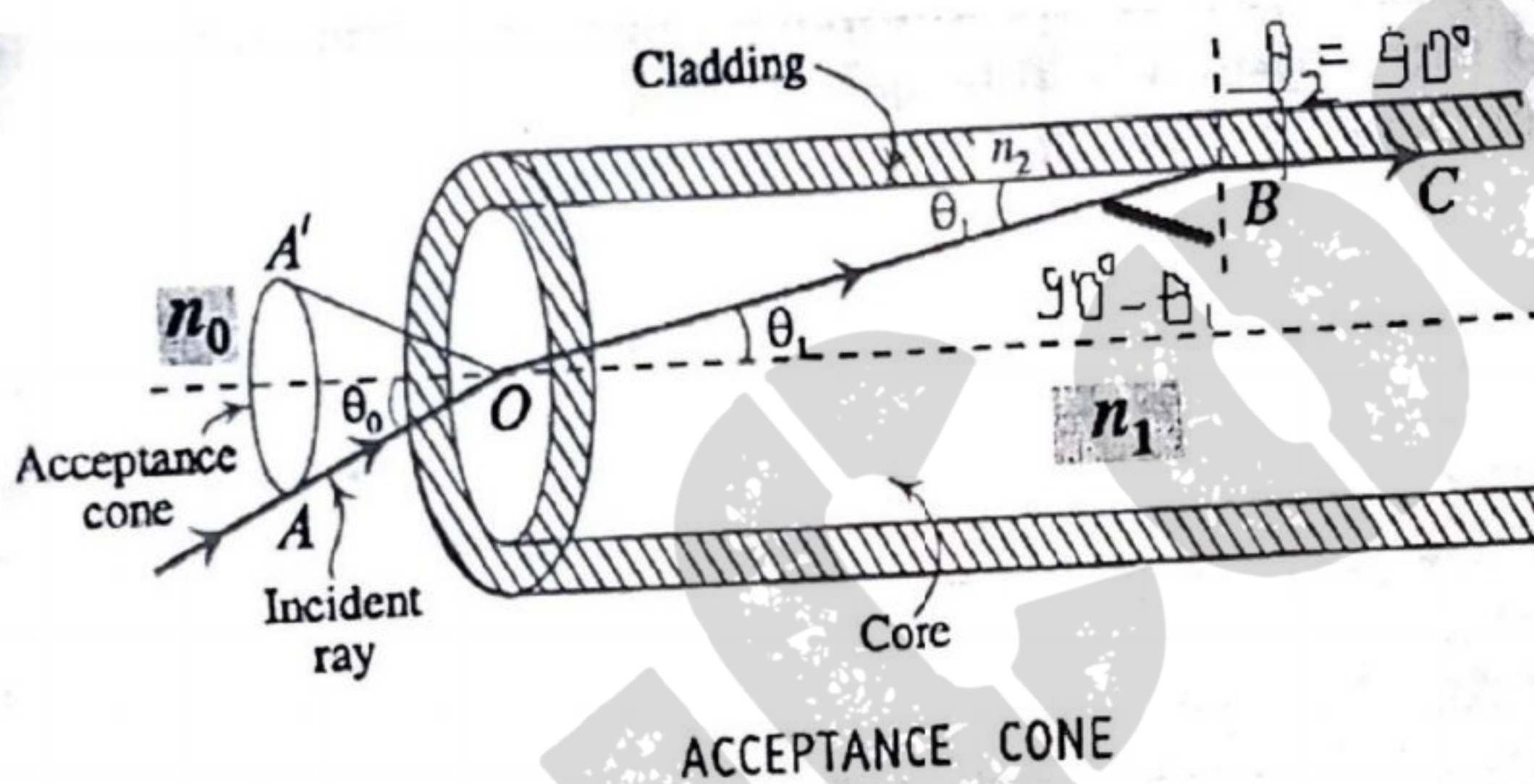
9. Laser beam is amplified by using optical resonators.

10. The laser output is 100 kW for continuous mode & 10 kW in pulsed mode.

5b. What is numerical aperture. Obtain the an ³¹ expression for numerical aperture in an optical fiber.

Numerical aperture:

The light gathering capability of an optical fiber & is known as numerical aperture.



Consider a light ray AO incident at an angle θ_0 enter into the fiber. Let θ_1 be the angle of refraction for the ray OB. The refracted ray OB incident at a critical angle $(90^\circ - \theta_1)$ at B grazes the interface between core & cladding along BC.

If the angle of incidence is greater than critical angle, it undergoes TIR. Thus θ_0 is called the waveguide acceptance angle & $\sin \theta_0$ is called the numerical aperture.

Let n_1 , n_2 & n_0 be the R.I. of core, cladding & surrounding medium resp.

Also OA incident ray, AB refracted ray at A, & θ_0 & θ_1 be the angle of incidence & angle of refraction at B resp.

By Snell's law at position 'O'.

Let n_0 , n_1 & n_2 be the R.I. of medium, core & cladding resp.

From Snell's law

$$n_0 \sin \theta_0 = n_1 \sin \theta_1 \quad \text{--- (1)}$$

From Snell's law

At Position 'B'

The angle of incidence = $(90^\circ - \theta_1)$

$$n_1 \sin (90^\circ - \theta_1) = n_2 \sin 90^\circ$$

$$n_1 \cos \theta_1 = n_2$$

$$\boxed{\cos \theta_1 = \frac{n_2}{n_1}} \quad \text{--- (2)}$$

From eqn (1), we have

Rearrange the eqn (1),

$$\sin \theta_0 = \frac{n_1}{n_0} \sin \theta_1$$

$$\sin \theta_0 = \frac{n_1}{n_0} \sqrt{1 - \cos^2 \theta_1} \quad \text{--- (3)} \quad (\sin \theta = \sqrt{1 - \cos^2 \theta})$$

Using eqn (2) in (3).

Put the value of $\cos \theta_1$ in equⁿ (3) from equⁿ (2),

$$\sin \theta_0 = \frac{n_1}{n_0} \sqrt{1 - \left(\frac{n_2^2}{n_1^2}\right)} = \frac{n_1}{n_0} \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$

$$\sin \theta_0 = \frac{n_1}{n_0} \times \frac{1}{n_1} \sqrt{n_1^2 - n_2^2}$$

$$\boxed{\sin \theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}} \quad \leftarrow (4)$$

If the surrounding medium is air $[n_0 = 1]$, then

$$\sin \theta_0 = \sqrt{n_1^2 - n_2^2}$$

where $\sin \theta_0 \rightarrow$ is called numerical aperture.

$$N.A = \sqrt{n_1^2 - n_2^2}$$

Therefore, for any angle of incidence equal to θ_i or less than θ_0

$$\theta_i < \theta_0$$

$$\sin \theta_i < \sin \theta_0$$

$$\sin \theta_i < \sqrt{n_1^2 - n_2^2}$$

5 c. A medium in thermal equilibrium at a temperature 300K has two energy levels with wavelength separation of $1\mu\text{m}$. Find the ratio of population densities of the upper & lower levels.

Given data:

Temp $T = 300\text{K}$

wavelength separation $\lambda = 1\mu\text{m} = 10^6\text{m}$

To find:

Ratio of population densities $\frac{N_2}{N_1} = ?$

We have Boltzmann factor,

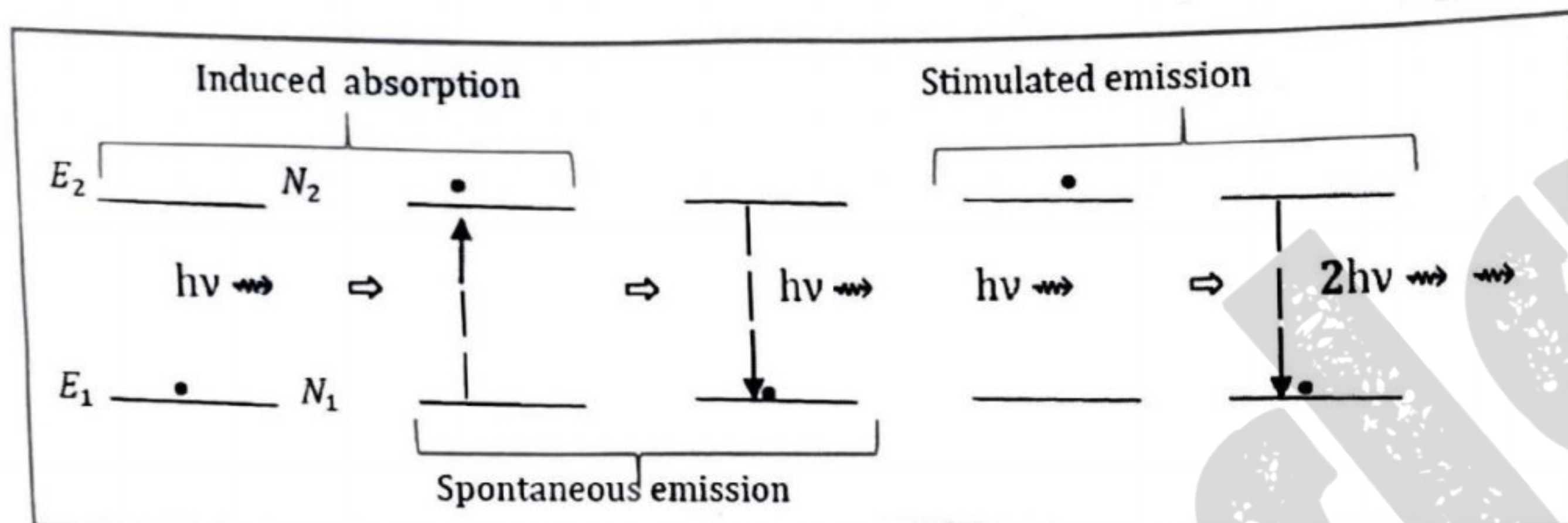
$$\begin{aligned}\frac{N_2}{N_1} &= e^{\frac{-\Delta E}{kT}} = e^{\frac{-h\nu}{kT}} = e^{\frac{-hc}{\lambda kT}} \\ &= e^{\frac{-\left(\frac{hc}{\lambda}\right)\left(\frac{1}{T}\right)}{k}} \\ &= e^{\frac{-0.014413}{\lambda T}} = e^{\frac{-0.014413}{10^6 \times 300}} \\ &= e^{-48.043}\end{aligned}$$

$$\frac{N_2}{N_1} = 1.365 \times 10^{-31}$$

$\therefore$ The ratio of population densities

$$\frac{N_2}{N_1} = 1.365 \times 10^{-31}$$

6a. Obtain an expression for energy density of radiation under thermal equilibrium condition in terms of Einstein's coefficients.



Consider a system of energy density E_ν in thermal equilibrium.

Consider two energy states E_1 & E_2 of a system of atoms ($E_2 > E_1$).

Let N_1 - number density of energy state E_1
& N_2 be the No. density of E_2

W.Kot

Rate of induced absorption = $B_{12} N_1 E_\nu$ — (1)
where B_{12} - Einstein's coefficients of induced absorption.

Rate of stimulated emission = $B_{21} N_2 E_\nu$ — (2)
where B_{21} - Einstein's coeff of stimulated emission

Rate of spontaneous emission = $A_{21} N_2$ — (3)
where A_{21} - Einstein's coeff for spontaneous emission.

At thermal equilibrium,

Rate of induced absorption = Rate of Spont. + Rate of Stimulated emission (4)

Using eqn (1), (2) & (3),

Eqn (4) becomes

$$B_{12} N_1 E_\nu = A_{21} N_2 + B_{21} N_2 E_\nu$$

$$E_\nu [B_{12} N_1 - B_{21} N_2] = A_{21} N_2$$

$$E_\nu = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

Dividing both N_1 & N_2 by $B_{21} N_2$, we get

$$E_\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12} N_1}{B_{21} N_2} - 1} \right] \quad \text{--- (5)}$$

According to Boltzmann's law

$$\frac{N_1}{N_2} = e^{\frac{E_2 - E_1}{kT}} = e^{\frac{h\nu}{kT}} \quad \text{--- (6)}$$

From eqn (5) & (6), we get

$$E_\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} e^{\frac{h\nu}{kT}} - 1} \right] \quad \text{--- (7)}$$

But, the energy density of black body radiation is given by Planck's law is

$$E_\nu = \frac{8\pi h \nu^3}{c^3} \left[\frac{1}{e^{\frac{h\nu}{kT}} - 1} \right] \quad \text{--- (8)}$$

Comparing eqn (7) & (8), we get

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3} \quad \& \quad \frac{B_{12}}{B_{21}} = 1$$

$$\boxed{B_{12} = B_{21}}$$

Thus coeff of induced absorption = coeff of stimulated emission.

66. What is attenuation? Discuss different types of attenuation in optical fibers.

Attenuation:

The power loss of optical signals, when they propagated in an optical fiber & is known as

Types of attenuation in an optical fiber:

1) Absorption loss:

Absorption of photons by impurities like metal ions such as iron, chromium, cobalt & copper in the silica glass of which the fiber is made. During signal processing photons interact with electrons of impurity atoms. The atoms are excited & de-excite by emitting photons of different characteristics. Hence it is a loss of energy. The other impurity such as hydroxyl ions (OH) causes significant absorption loss. The ~~impur~~ absorption of photons by fiber material itself is called intrinsic absorption.

2. Scattering loss:

When the wavelength of the photon is comparable to the size of the particle then the scattering takes place. Because of the non-uniformity in manufacturing, the R.I. changes with length leads to a scattering. This type of scattering is called as Rayleigh-scattering. It is ^{inversely} proportional to λ^4 .

3. Radiation losses:

Radiation losses due to macroscopic bends & microscopic bends.

a. Macroscopic bending:

All optical ~~bending~~ fibers are having critical radius of curvature provided by the manufacturer. If the fiber is bent below that specification of radius of curvature, the light ray incident on the core-cladding interface will not satisfy the condition of TIR. This causes loss of optical power.

b. Microscopic bending:

Optical power loss in optical fibers is due to non uniformity of the optical fibers when they are laid. Non uniformity is due to manufacturing defects & also lateral pressure built up on the fiber. The defect due to non-uniformity (microbending) can be overcome by introducing optical fibers inside a good strengthened polyurethane jacket.

60. A step-index optical fiber with core diameter of 60 μm & core & cladding R.I's of 1.50 & 1.48 respectively, when the wavelength of 850 nm is propagating through it. Calculate the numerical aperture, fractional index change, V-number & number of modes in the fiber.

Given:

Diameter of core $d = 60 \mu\text{m} = 60 \times 10^{-6} \text{m}$

R.I. of core $n_1 = 1.50$

R.I. of cladding $n_2 = 1.48$

wavelength $\lambda = 850 \mu\text{m} = 850 \times 10^{-6} \text{m}$

Calculate: Numerical aperture, $NA = ?$

fractional index change $\Delta = ?$

V-number $V = ?$

Number of modes $N = ?$

$n_0 = 1$

(i) Numerical aperture $NA = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$

$$= \frac{\sqrt{1.50^2 - 1.48^2}}{1}$$

$$NA = 0.244$$

(ii) Fractional index change, Δ

$$\Delta = \frac{n_1 - n_2}{n_1} = \frac{1.50 - 1.48}{1.50} = 0.0133$$

(iii) V number $= V = \frac{\pi d}{\lambda} \sqrt{n_1^2 - n_2^2}$

$$= \frac{3.142 \times 60 \times 10^{-6}}{850 \times 10^{-6}} \times 0.244 = 45.998$$

(iv) Number of modes $= N = \frac{V^2}{2} =$

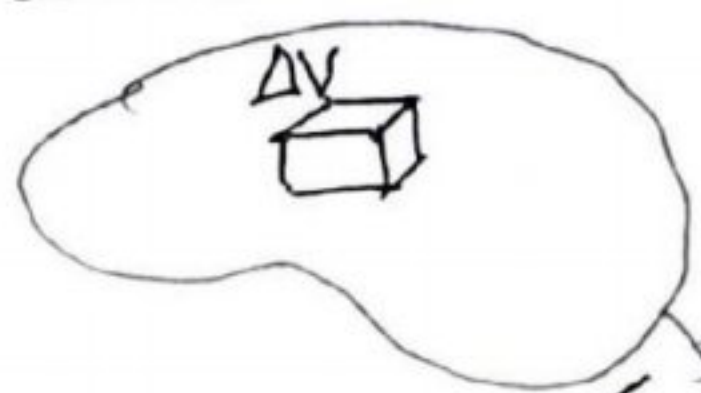
7a. State and prove Gauss Divergence theorem.

Statement:-

The integral of the normal component of the ~~flux~~ flux density over any closed surface in an electric field is equal to the volume integral of the divergence of the flux throughout the space enclosed by the surface. It is represented mathematically as

$$\oint_S \vec{D} \cdot d\vec{S} = \int_V \nabla \cdot \vec{D} dv$$

Proof:-



Gaussian Surface

Consider a gaussian surface in a region with certain charge density (Fig.) Inside the surface, consider a differential volume element

" ΔV ". Let ΔQ be the charge within the element

If " ρ " is the charge density & since ρ can vary continuously in the volume, we have

$$\rho_v = \lim_{\Delta V \rightarrow 0} \left[\frac{\Delta Q}{\Delta V} \right] = \frac{dQ}{dV} \quad \text{--- (1)}$$

$$\therefore dQ = \rho_v dv$$

If Q is the total charge enclosed by the Gaussian surface, then

$$Q = \int_V dq = \int_V \rho_v dv \quad \text{--- (2)}$$

But, w.k.t. $\nabla \cdot \vec{D} = \rho_v$ (By Maxwell's first equation).

$$Q = \int_V \nabla \cdot \vec{D} dv \quad \text{--- (3)}$$

Now, by applying Gauss' law to the Gaussian surface, we have

$$\oint_S \vec{D} \cdot d\vec{S} = Q \quad \text{--- (4)}$$

$\therefore$ By equⁿ (3) & (4), we have

$$\boxed{\oint_S \vec{D} \cdot d\vec{S} = \int_V \nabla \cdot \vec{D} dv} \quad \text{--- (5)}$$

This is the Gauss's divergence theorem
or simply the divergence theorem.

7b. Derive wave equation in terms of electric field using Maxwell's equation for free space.

Wave eqn in differential form in free space in terms of Field.

Let us consider the two curl eqns of Maxwell.

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

But, w.k.t, $D = \epsilon E$ & $B = \mu H$

$$\nabla \times \vec{H} = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad \text{--- (1)}$$

and.

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{--- (2)}$$

Let us obtain the wave equation in terms of the electric field, for which we must obtain an equation relating the spatial co-ordinates of $\vec{E}$ to its time co-ordinates. This is done by eliminating $\vec{H}$ between the two equations in the following way.

Taking curl for both sides of eqn (2), we have

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t} (\nabla \times \vec{H}) \quad \text{--- (3)}$$

As per vector analysis,

$$\begin{aligned}\nabla \times \nabla \times \vec{E} &= \nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} \\ &= \nabla \left(\frac{\rho_v}{\epsilon} \right) - \nabla^2 \vec{E} \quad \text{--- (4)}\end{aligned}$$

[Since, as per Maxwell's equation,
 $\nabla \cdot \vec{D} = \rho_v \quad \text{or} \quad \nabla \cdot \vec{E} = \frac{\rho_v}{\epsilon}$]

$\therefore$ From equⁿ (3) & (4)

$$\nabla \left(\frac{\rho_v}{\epsilon} \right) - \nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t} [\nabla \times \vec{H}]$$

Using equⁿ (1) in the right side, we have

$$\nabla \left(\frac{\rho_v}{\epsilon} \right) - \nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t} \left[\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \right]$$

$$\text{or} \quad \nabla^2 \vec{E} = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = \mu \frac{\partial \vec{J}}{\partial t} + \nabla \left(\frac{\rho_v}{\epsilon} \right) \quad \text{--- (5)}$$

Hence equⁿ (5) represents the wave equⁿ in $\vec{E}$ for a medium with constant μ & ϵ . i.e. a homogeneous & isotropic medium.

If we consider free space, i.e. space where there is no charges or currents then $\rho_v = 0$ & $\vec{J} = 0$ & equⁿ (5) becomes,

$$\boxed{\nabla^2 \vec{E} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0} \quad \text{--- (6)}$$

7c.

Prove that

$$3y^4z^2\vec{a}_x + 4x^3z^2\vec{a}_y + 3x^2y^2\vec{a}_z \text{ is}$$

Solenoidal.

The given vector field is $\vec{A}$

$$\vec{A} = 3y^4z^2\vec{a}_x + 4x^3z^2\vec{a}_y + 3x^2y^2\vec{a}_z \quad \text{--- (1)}$$

Writing the general formula for $\vec{A}$ we have

$$\vec{A} = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z \quad \text{--- (2)}$$

Comparing eqn (1) & (2), we have

$$A_x = 3y^4z^2 \quad A_y = 4x^3z^2 \quad A_z = 3x^2y^2$$

The respective partial derivatives

$$\frac{\partial A_x}{\partial x} = 0 \quad \frac{\partial A_y}{\partial y} = 0 \quad \frac{\partial A_z}{\partial z} = 0$$

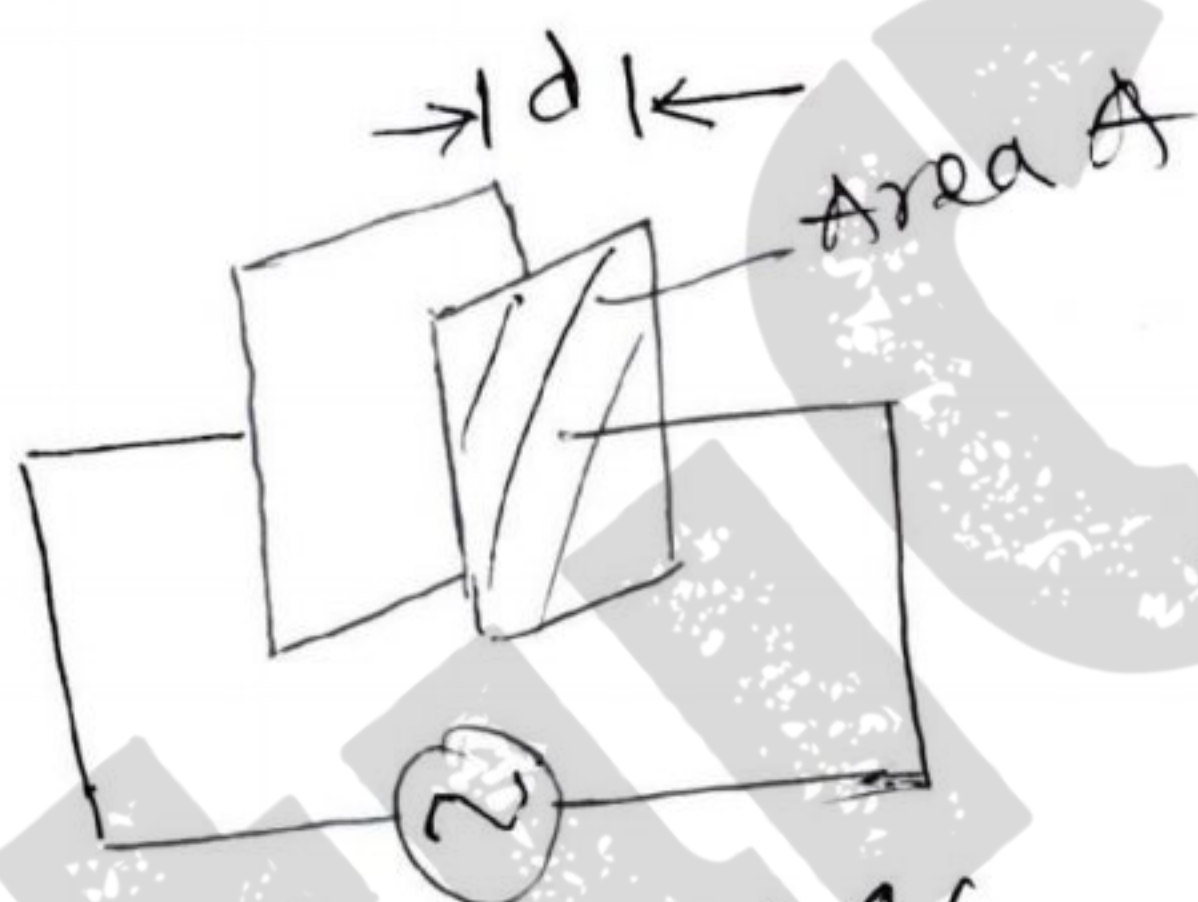
For $\vec{A}$ to be solenoidal $\nabla \cdot \vec{A} = 0$.

8 a. What is displacement current? Derive the expression for displacement current.

Displacement current:

Displacement current is the correction factor in Maxwell's equation that appears in time-varying condition but doesn't describe any movement of charges though it has an associated magnetic field.

Expression for Displacement current:



Consider a parallel plate capacitor connected across an a.c source. Let the area of each of the plates be A .

Capacitor across A.C.

c) Conduction current:

Let the potential be $V = V_0 e^{i\omega t}$

we have, $D = \epsilon E$

For a parallel plate capacitor, $E = \frac{V}{d}$

$$D = \frac{\epsilon V}{d} = \frac{\epsilon V_0 e^{i\omega t}}{d}$$

The displacement current density is given by

$$\vec{D} = \epsilon \vec{E}$$

$$\left(\frac{\partial D}{\partial t} \right)$$

If I_D is the displacement current, then

$$\left(\frac{\partial D}{\partial t}\right) = \frac{I_D}{A}$$

$$\therefore I_D = \left(\frac{\partial D}{\partial t}\right) A$$

$$= \frac{\partial}{\partial t} \left[\frac{\epsilon}{d} V_0 e^{j\omega t} \right] A,$$

or displacement current I_D ,

$$I_D = \frac{j\omega \epsilon A}{d} V_0 e^{j\omega t}$$

(6)

5b. Explain Faraday's Laws of Electromagnetic induction, Ampere's law & express them in the same in point form.

Faraday's law:

Faraday's law can be stated as "the magnitude of the induced emf in a circuit is equal to the rate of change of the magnetic flux through it, & its direction opposes the flux change."

∴ the induced emf e is expressed as

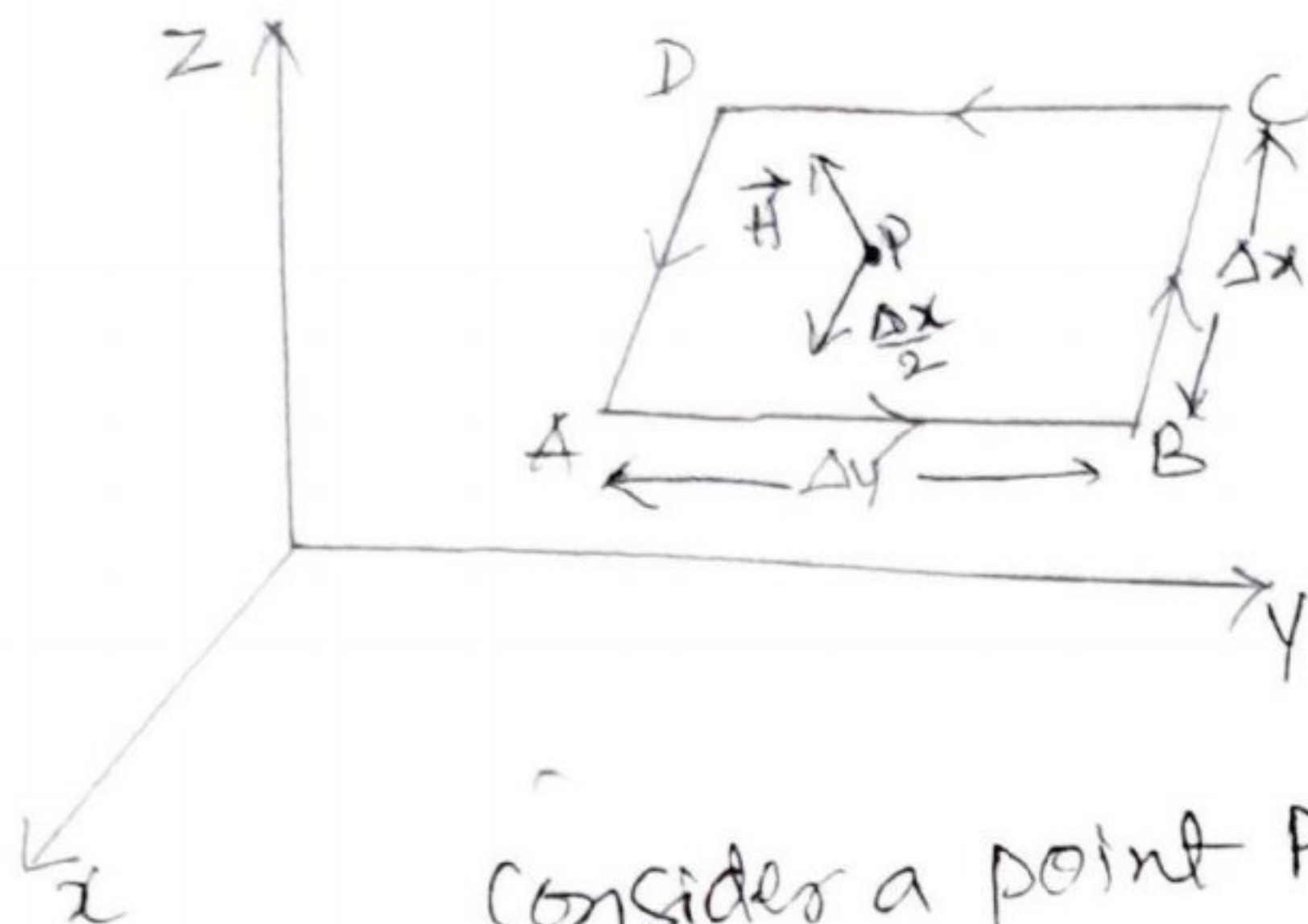
$$e = - \frac{d\phi}{dt} \quad \text{--- (1)}$$

where, ϕ is the flux linkage with the circuit such as a turn in a conducting coil.

If we consider a coil with N turns, then the emf induced across the coil is given by

$$e = - N \frac{d\phi}{dt} \quad \text{--- (2)}$$

Ampere's law:



Consider a point P in a magnetic field $\vec{H}$ (Fig 1). In order to visualise the curl of the magnetic field, imagine a rectangular loop ABCD around P in a plane parallel to x-y plane.

If J_z & $\vec{a}_z$ are the current density & unit vector resp. along z-direction, then by finding the line integral of $\vec{H} \cdot d\vec{l}$ over the closed path ABCDA, it is possible to show that,

$$(\text{curl } \vec{H})_z = J_z \vec{a}_z$$

where the subscript - z for curl $\vec{H}$ signifies that the curl is in a plane parallel to x-y plane.

Similarly for the curl in a plane parallel to y-z plane, we have

$$(\text{curl } \vec{H})_x = J_x \vec{a}_x$$

and for the curl in a plane parallel to x - z plane, it will be

$$[\text{curl } \vec{H}]_3 = J_y \vec{a}_y$$

Here, $[\text{curl } \vec{H}]_1$, $[\text{curl } \vec{H}]_2$ & $[\text{curl } \vec{H}]_3$ are the components of curl of $\vec{H}$ around the point P .

But curl $\vec{H}$ is represented as $\nabla \times \vec{H}$

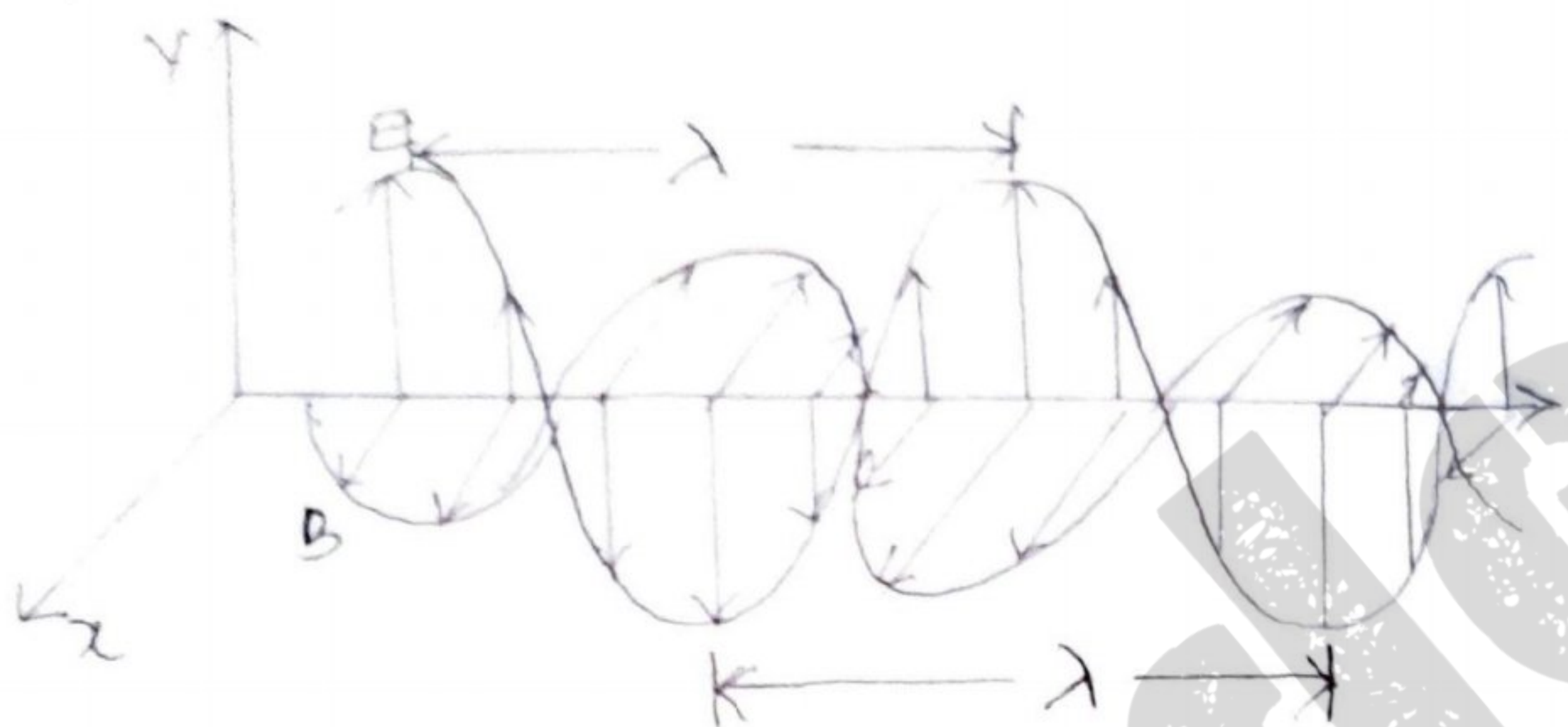
$$\nabla \times \vec{H} = J_x \vec{a}_x + J_y \vec{a}_y + J_z \vec{a}_z$$

$$\text{or } \nabla \times \vec{H} = \vec{J} \quad \text{--- (II)}$$

where, $\vec{J}$ is the current density vector with components J_x , J_y & J_z along the 3 cartesian co-ordinates.

Equⁿ (II) is the ampere's circuital law for time-invariant fields & it is another Maxwell's equation.

3 c. Elucidate the Transverse nature of EM waves through Linear polarization.

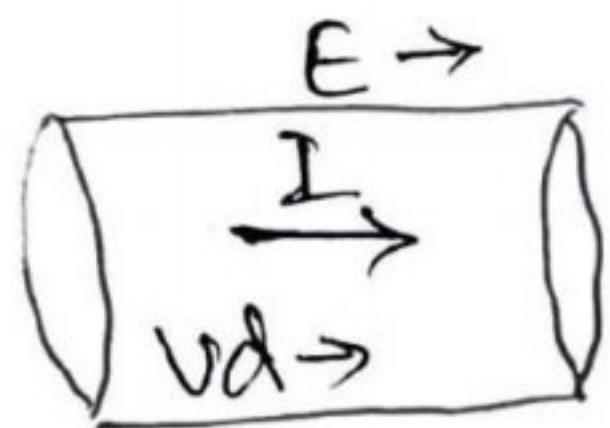


An electromagnetic wave comprises of varying electric & magnetic fields in a direction $\perp$ to the direction of propagation. In (Fig 1), variations in the electric field E and the magnetic field B are shown respectively along x and y -axes, whereas the direction of propagation is shown along z -axis:

The field strength variation of either of them is of the pattern of a wave, & each of them has same wavelength. At no time, there could be only an electric wave, or only a magnetic wave. The electromagnetic wave - duo ~~to~~ always appear together as a combined entity as electromagnetic wave.

The wave as a whole is carried bodily along the line of intersection of the two planes comprising of E & B variations. The velocity of movement of any electromagnetic wave, irrespective of wavelength or intensity is same in a given medium.

9 a. Derive an expression for electrical conductivity in extrinsic & intrinsic semiconductors



Consider a semiconductor of area of cross section A , carrying current I .

Let v_d be the drift velocity of charge carriers.

In the presence of electric field, both electrons & the holes contribute to the conductivity of the semiconductor. $I = NeAv_d$ — (1)

Let us consider first the conductivity in a semiconductor is due to the flow of electrons only.

The expression for current flows due to free e^- as well as holes

Current due to e^- can be written as

$$I_e = NeAv_e \quad \text{--- (2)}$$

where I_e → current in a conductor due to electrons.

e → charge on e^-

A → surface area of the semiconductor

v_e → drift velocity of e^- .

N_e → No. density of e^-

Similarly, current due to holes is

$$I_h = N_h e A V_h \quad \text{--- (3)}$$

where $V_h \rightarrow$ drift velocity of holes

$N_h \rightarrow$ No. density of holes

Total current in semiconductor is

$$I = I_e + I_h \quad \text{--- (4)}$$

Using eqn (2) & (3), we have

$$I = N_e e A V_e + N_h e A V_h$$

$$I = e A [N_e V_e + N_h V_h]$$

$$\frac{I}{A} = e [N_e V_e + N_h V_h]$$

$$J = e [N_e V_e + N_h V_h] \quad \text{--- (5)}$$

Rel. betⁿ J , σ & E is

$$J = \sigma E$$

Eqn (5) becomes

$$\sigma E = e [N_e V_e + N_h V_h]$$

Dividing by E on both sides, we get

$$\sigma = e \left[N_e \left(\frac{V_e}{E} \right) + N_h \left(\frac{V_h}{E} \right) \right]$$

$$\sigma = e [N_e \mu_e + N_h \mu_h] \quad \text{--- (6)}$$

where $\mu_e = \frac{V_e}{E}$

and

$$\mu_h = \frac{V_h}{E}$$

This is eqn for conductivity in SC's.

Conductivity in intrinsic semiconductor.

In the case of intrinsic SCs, then the no. of e^- are equal to no. of holes

For intrinsic semiconductors: ($N_h = N_e$)

$N_i \rightarrow$ number density of intrinsic charge carrier

$$\sigma = e [N_e \mu_e + N_h \mu_h]$$

For intrinsic SCs $\rightarrow N_h = N_e$

$$\sigma = N_i e [\mu_e + \mu_h] \quad \text{--- (7)}$$

For extrinsic semiconductor: $N_h \neq N_e$

For N-type semiconductor $N_e > N_h$

$$\sigma = N_e e \mu_e \quad \text{--- (8)}$$

$N_e \rightarrow$ density of donor impurity atoms.

For P-type semiconductor: $N_h > N_e$

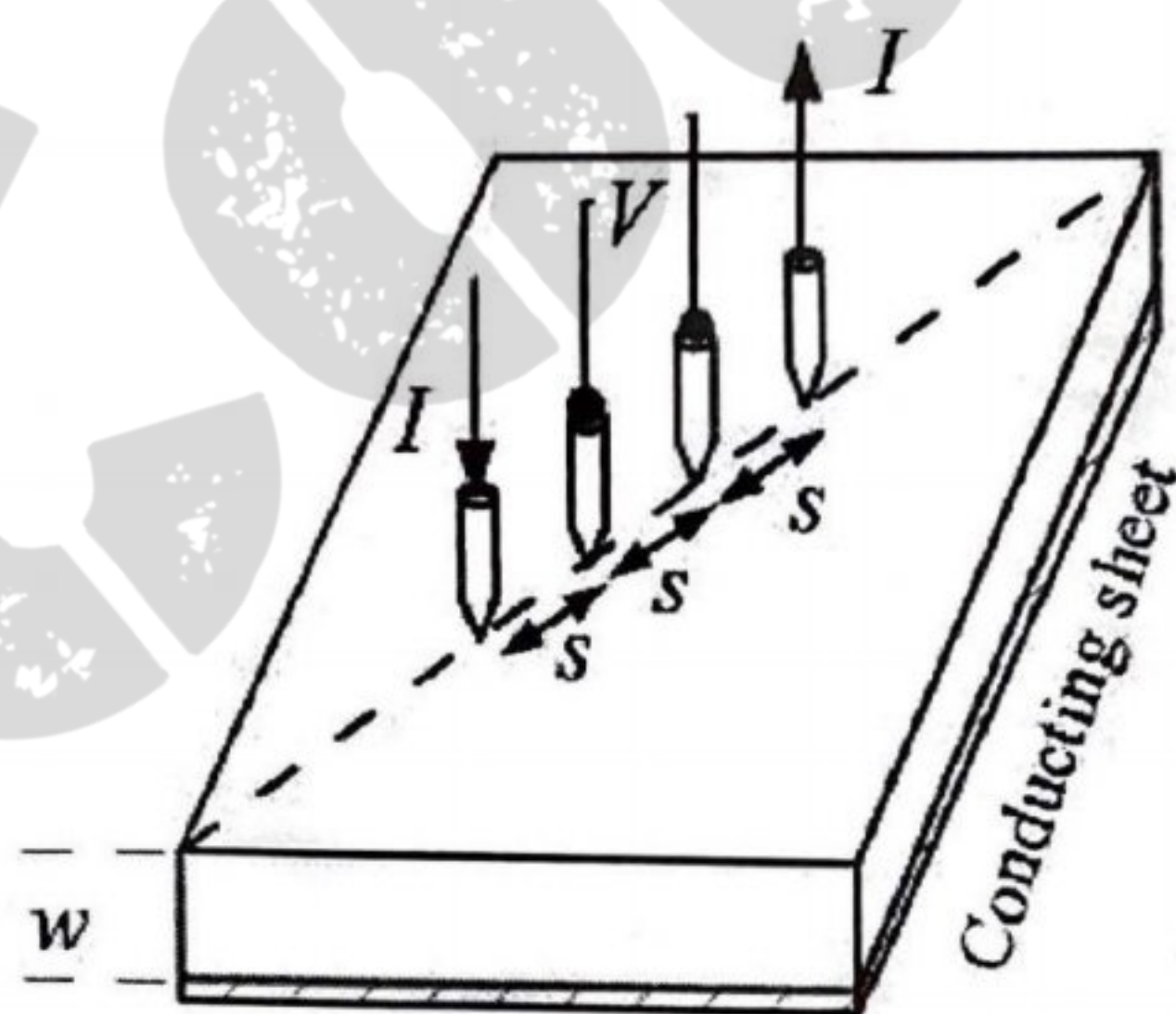
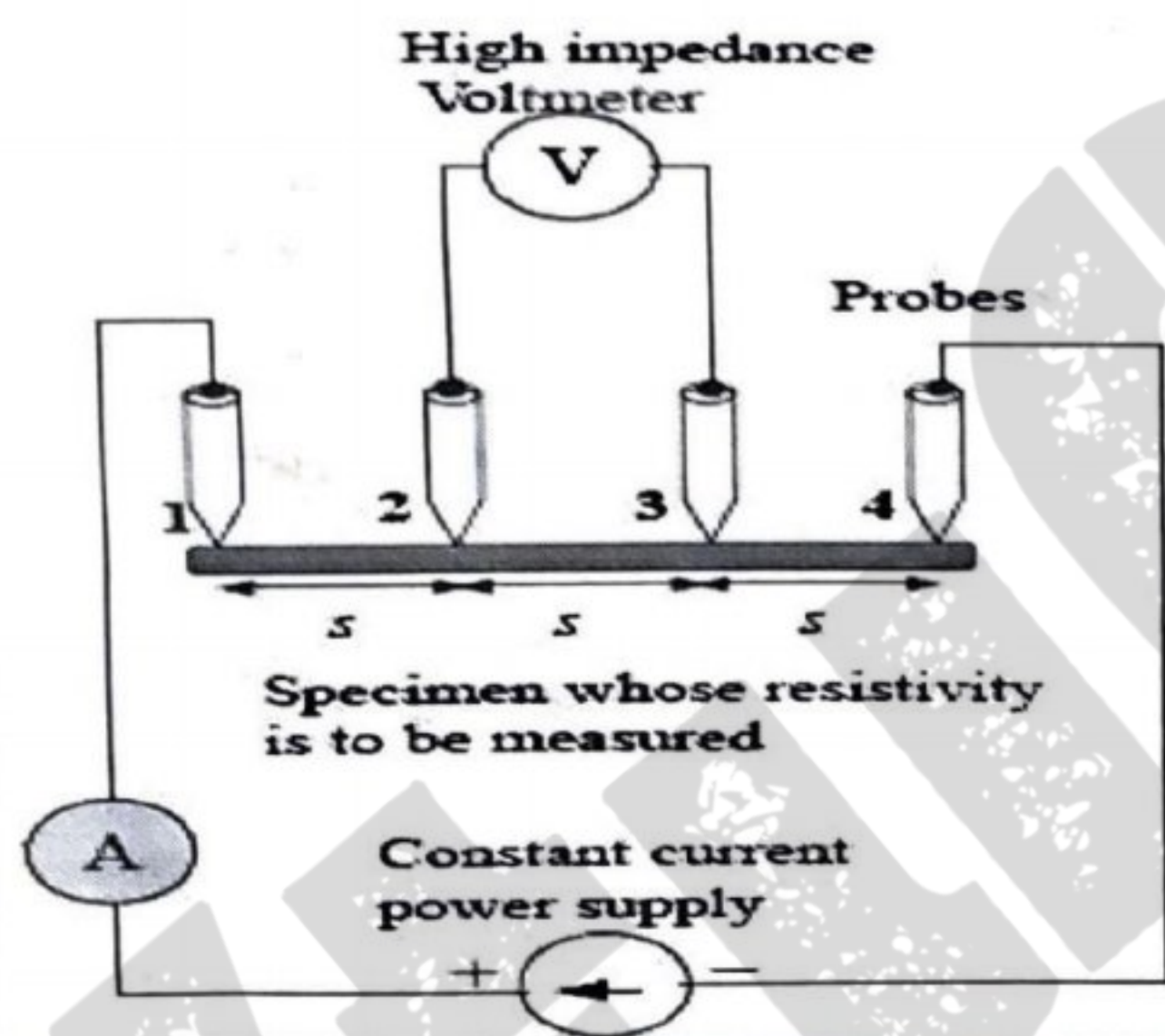
$$\sigma = N_h e \mu_h \quad \text{--- (9)}$$

$N_h \rightarrow$ density of acceptor impurity atoms.

9b. Explain how the resistivity of a semiconductor is determined using four probe method? Mention any two applications of four - probe method.

Apparatus:

The expt. set up consists of probe arrangement, sample, oven $0-200^{\circ}\text{C}$, constant current generator, oven power supply & digital panel meter [measuring voltage & current]



Procedure:

1. Select the semiconductor sample from the Combo box.
 2. Select the source current from the slider. Adjust the slider based on the range of current.
 3. Select the range of oven from the Combo box.
 4. Set the temp. from the slider
- 5.

5. Click on the -Run button to start heating the oven for a particular ~~test~~ interval from the default 25°C to the temp that we set.

6. Click on the set bottom to display the temp that we set in the oven.

7. Click on the measure button to display the present temp in the oven

8. Select the range of voltmeter from the combo box.

9. Measure the voltage using voltmeter.

10. Calculate the resistivity of Si in $\Omega\text{-cm}$.

11. Plot the graph. Temp along x-axis & ρ of Si along y-axis.

$$\rho = \frac{\rho_0}{f\left(\frac{w}{s}\right)}$$

$f(w/s)$ is a division of computing resistivity which depends on the value of w/s .

$$\rho_0 = \frac{V}{I} \cdot 2\pi s$$

$V \rightarrow$ p.d. across the inner probes in volts.

$I \rightarrow$ Current through the outer pair of probes in ampere.

$s \rightarrow$ Spacing between the probes in meters.

9c. An n-type Germanium sample as donor density of $10^{21}/m^3$, it is arranged in a Hall experiment having magnetic field of $0.5T$ and the current density is $500 A/m^2$. Find the Hall voltage if the sample is $3mm$ wide.

Given data:

number density $n = 10^{21}/m^3$

magnetic field $B = 0.5T$

current density $I = 500 A/m^2$

Hall voltage $V_H = ?$

thickness $d = 3mm = 3 \times 10^{-3} m$.

Hall voltage, V_H is given by

$$V_H = \frac{IBd}{nqA} = \frac{I}{A} \cdot \frac{Bd}{nq}$$

$$= \frac{IBd}{nq} = \frac{500 \times 0.5 \times 3 \times 10^{-3}}{10^{21} \times 1.6 \times 10^{19}}$$

$$= \frac{250 \times 10^{-5} \times 3}{1.6 \times 100}$$

$$= 4468.75 \times 10^{-7}$$

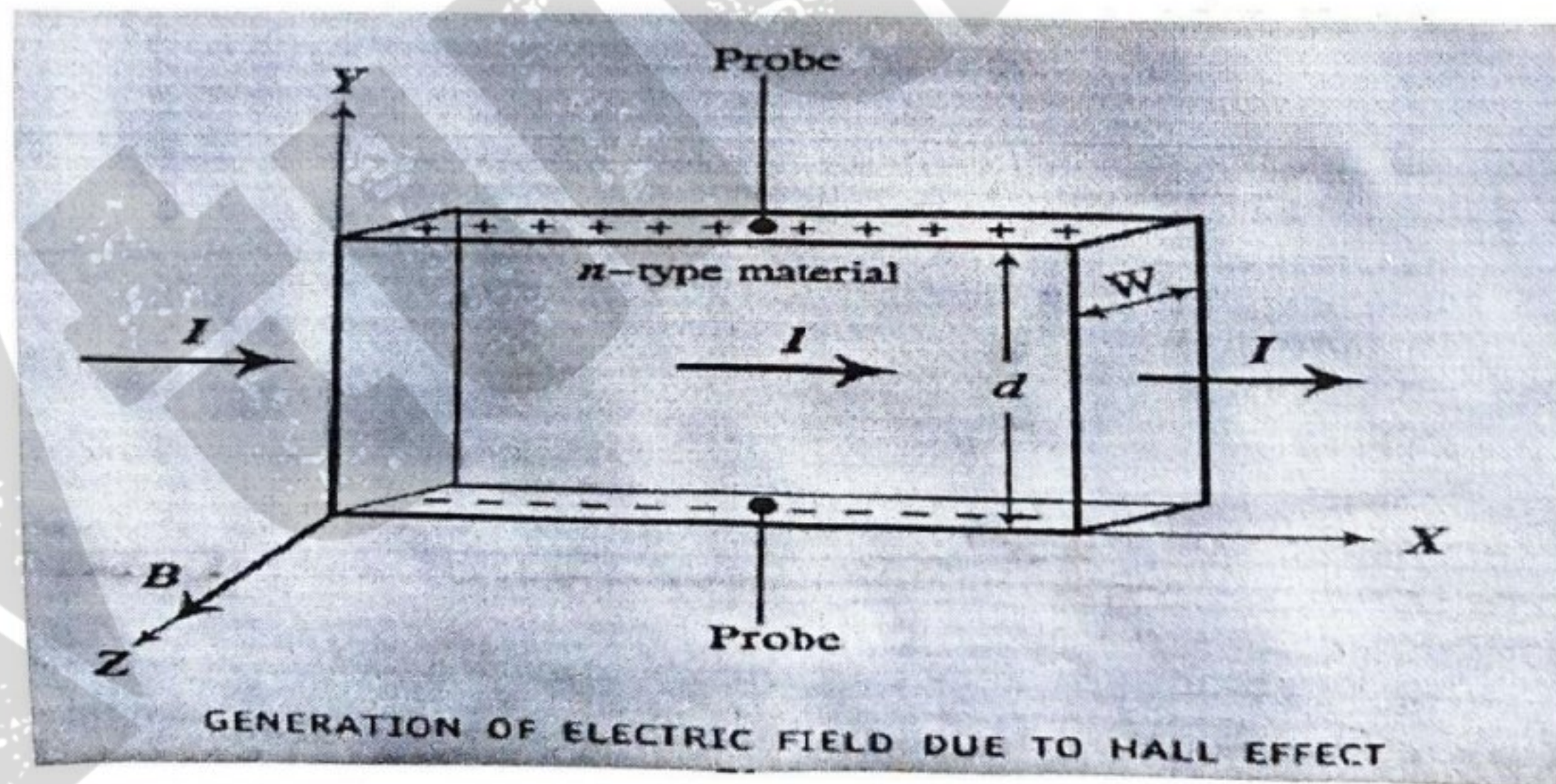
$$V_H = 0.446875 \times 10^{-4} V$$

10 a. What is Hall effect? Obtain an expression for the Hall coefficient.

Statement:

When a current carrying semiconductor is subjected to external magnetic field $\perp$ to the direction of current. The electric field induced (produced) across the material is in a direction $\perp$ to both the magnetic field & the direction of current flow. This phenomenon is called Hall effect.

Expression for Hall coefficient:



Consider a rectangular n-type semiconductor material in which the current I is flowing in the x -direction.

Let the semiconducting material be of n-type, which means that the charge carriers are electrons.

Let the magnetic field (B) be applied along the z -direction.

Therefore, the electrons experience the Lorentz force (F_L) along the $-y$ -direction given by

$$F_L = -Bev \quad \text{--- (1)}$$

where $e \rightarrow$ magnitude of charge on e^- & $v \rightarrow$ is the drift velocity.

$\therefore$ The e^- density increases on lower surface & in upper surface becomes $+ve$ charged due to deficiency of e^- . Hence the potential developed between two surfaces called Hall voltage, which produces an electric field E_H & Hall field (E_H).

The e^- exerts an upward force F_H due to the Hall field & is given by

$$F_H = -eE_H \quad \text{--- (2)}$$

Equating eqn (1) & (2), we have

$$F_L = F_H$$

$$-eE_H = -Bev$$

$$E_H = Bv \quad \text{--- (3)} \quad v \rightarrow \text{drift velocity}$$

If 'd' is the distance between upper and lower surface of the slab, then,

$$E_H = \frac{V_H}{d}$$

$$\boxed{V_H = E_H \cdot d} \text{ from eqn (3).}$$

$$\boxed{V_H = B V d} \text{ --- (4)}$$

The current density $J = \frac{I}{A} = \frac{n e A V}{A} = n e V = p V$ --- (5)

$$J = p V \text{ --- (5)}$$

$\therefore p = n e$ p be concentration of charge carriers.

For the given semiconductor,

$$J = \frac{I}{A} = \frac{I}{w d}$$

$w \rightarrow$ thickness of the material,

$$\therefore A = w \cdot d$$

Equating (5) & (6) $p V = \frac{I}{w d}$

$$\boxed{V = \frac{I}{p w d}} \text{ --- (7)}$$

Substituting eqn (7) in (4).

$$V_H = B \left(\frac{I}{p w d} \right) d$$

$$\boxed{V_H = \frac{B I}{p w}} \text{ --- (8)}$$

$$\boxed{p = \frac{B I}{V_H w}}$$

Thus, by measuring V_H , I & w & by knowing B , the charge density p can be determined.

10/6

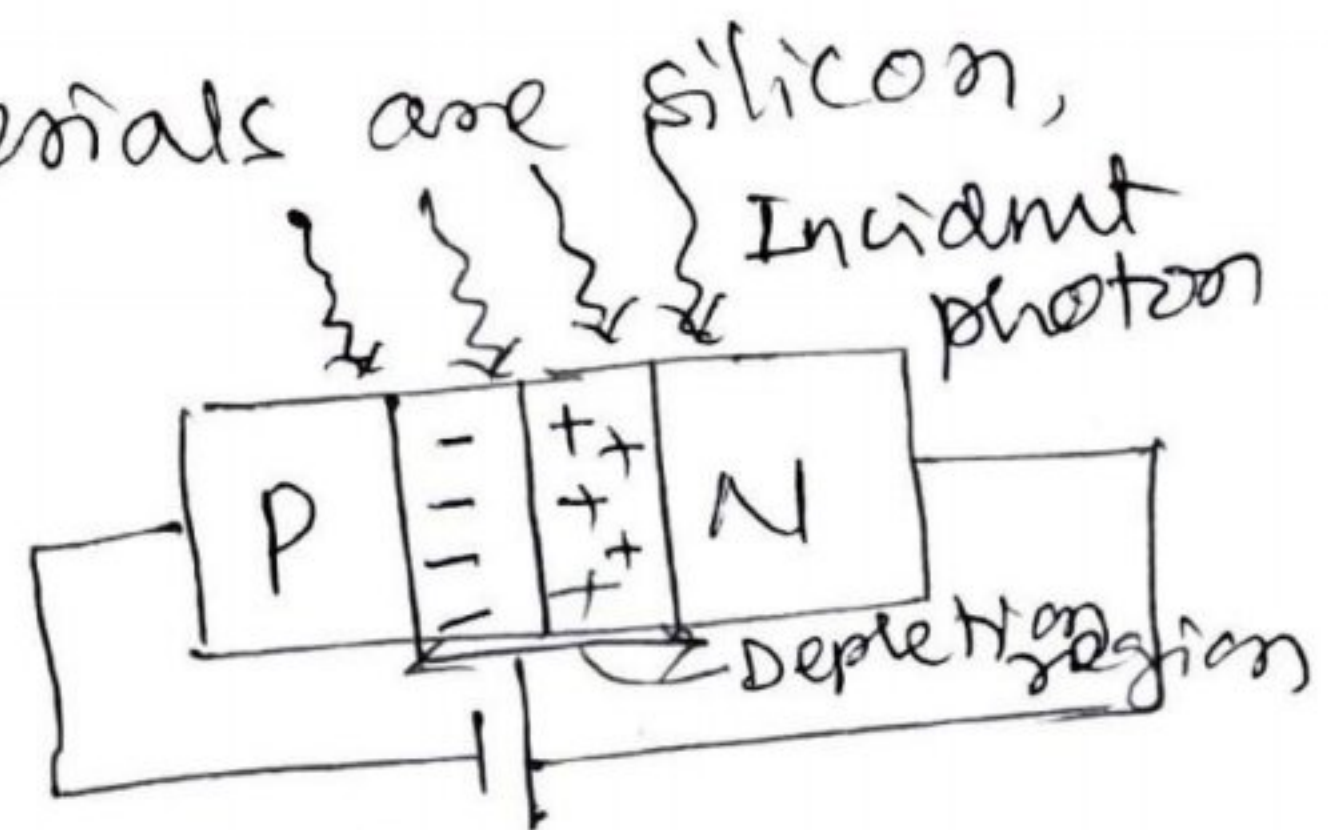
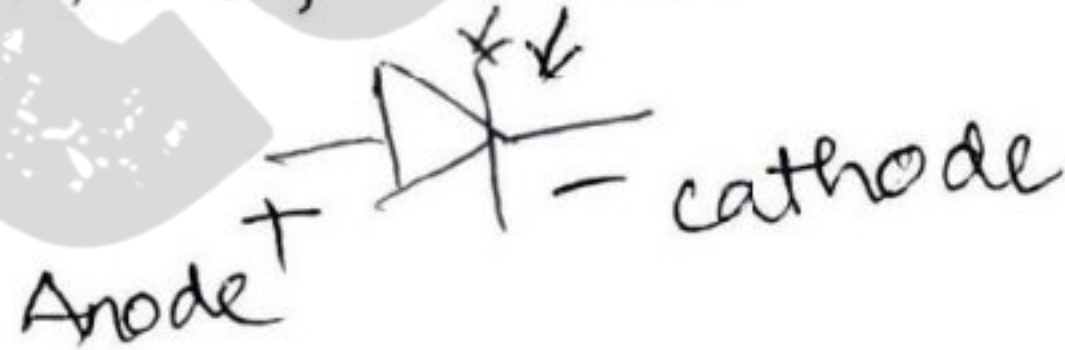
Explain the construction & working of a photodiode? Discuss the power responsivity in a photodiode.

Construction:

It is a form of light sensor ~~or~~ that converts light energy into electrical energy (voltage or current). Photodiode is a type of semi conducting device with PN junction. Between the p (+ve) & n (-ve) layers, an intrinsic layer is present. The photo diode accepts light energy as input to generate electric current. It is also called as photodetector, photo sensor or light detector. Photodiode operates in reverse bias condition i.e. p side is connected to -ve terminal of the battery & n-side to the +ve terminal of battery.

Typical photodiode materials are silicon, Ge, In-Ga-As & InGa-As

Symbol



Working:

A photo diode is PIN structure or p-n junction. When a photon of sufficient energy strike the diode, it creates an electron-hole pair. This mechanism is also known as the inner photoelectric effect.

Photodiode responsivity:

The responsivity of silicon photodiode is a measure of light. It is defined as ratio of photocurrent (I_p) to incident light power P at given wavelength.

$$\text{Responsivity, } R_\lambda = \frac{I_p}{P}$$

In other words, it is measure of the effectiveness of conversion of light power into electric current. It varies with the wavelength of incident light, applied reverse bias and temperature.

10
C.

The resistivity of intrinsic Ge at 27°C is equal to $0.47 \Omega\text{-m}$. Assuming electron & hole mobilities as $0.38 \text{ m}^2/\text{V/s}$ & $0.18 \text{ m}^2/\text{V/s}$ resp. Calculate the intrinsic carrier density.

Given data:

Resistivity $\rho = 0.47 \Omega\text{-m}$

$$e = 1.6 \times 10^{-19} \text{ C}$$

e mobility, $\mu_e = 0.38 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$

holes mobility, $\mu_h = 0.18 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$

intrinsic carrier density $n_i = ?$

conductivity in ^{intrinsic} semiconductor eqn is

$$\sigma_{\text{intrinsic}} = n_i e (\mu_e + \mu_h)$$

$$\text{Resistivity } \rho_i = \frac{1}{\sigma_i}$$

$$\rho_i = \frac{1}{n_i e (\mu_e + \mu_h)}$$

$$n_i = \frac{1}{\rho_i e (\mu_e + \mu_h)}$$

$$n_i = \frac{1}{0.47 \times 1.6 \times 10^{-19} (0.38 + 0.18)} = \frac{10^{19}}{0.47 \times 1.6 \times 0.56} = \frac{10^{19}}{0.42112}$$

$$n_i = 2.374 \times 10^{19} / \text{m}^3$$



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