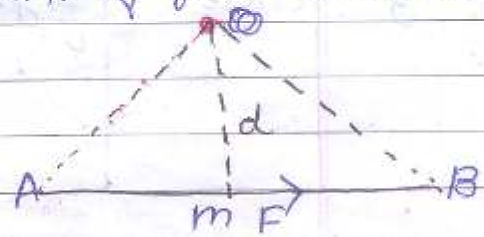
UNIT - 2nd

→ Location of centeriod & centro gravity are moment of inertia
⇒ mass moment of inertia
→ Law of machines
→ Variation of mechanical advantages, efficiency, reversibility of M/C
→ Pulleys & axle wheel & differential axele
⇒ Transmission of power through belt & Rope

NOTE :- To locate force graphically select a suitable scale and draw a line parallel to the line of action of given force, the length of line must be equal to magnitude of force. By the definition of force about OE



∴ moment $M_o = F \times d$

$M_o = AB \times OM$

The area of triangle $\Delta OAB = \frac{1}{2} \times AB \times OM$

$\boxed{2\Delta OAB = AB \times OM}$

$\boxed{2\Delta OAB = M_o}$

The moment P about D

$\boxed{M_P = 2 ODC}$

The moment of Q about D

$\boxed{M_Q = 2 ODA}$

The moment of R about D is $\boxed{M_R = 2 ODB}$

$2ODB = 2ODA + 2OAB$, But the geomentry area of given figure three triangles are same

$\Delta OAB, \Delta OBC, \Delta ODC$ put the value of Δ

$\Delta OAB = \Delta ODC$

$2\Delta ODB = 2ODC + 2ODA$

$\boxed{M_R = M_P + M_Q}$

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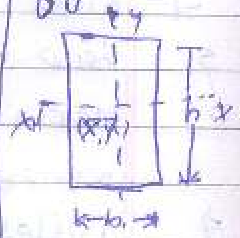
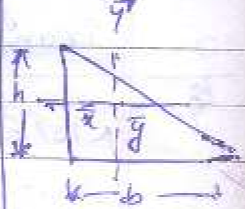
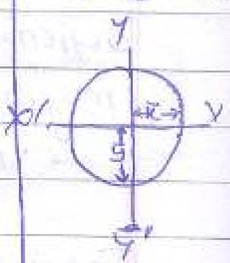
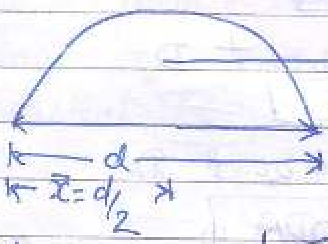
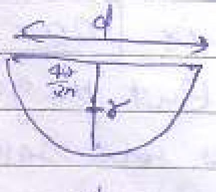
UNIT- IInd

Centroids of plane geometrical figure :-

S.No	Shape	Area	$\overline{X}$	$\overline{Y}$	figure
1	Rectangle	$b \times h$	$b/2$	$h/2$	
2	Triangle	$\frac{1}{2} \times b \times h$	$b/3$	$h/3$	
3	Circle	$\frac{\pi}{4} d^2$	$d/2$	$d/2$	



$\bar{y} = \frac{4r}{3\pi}$



$\bar{x} = d/2$

$\bar{y} = r - \frac{4r}{3\pi}$

$\bar{y} = d/2$

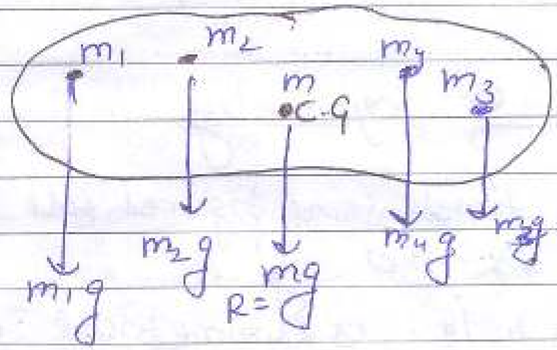
$\bar{x} = r - \frac{4r}{3\pi}$

9/3/16

CENTRE OF GRAVITY & CENTROID:-

The centre of gravity is defined as the point in the body through which the resultant of all these parallel forces of attraction formed by weight of the body passes.

⇒ It is the usually denoted by C.G. or simply "G"

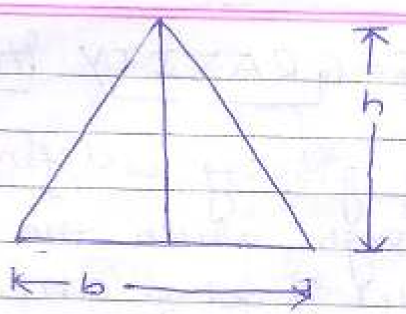


NOTE :- Everybody has ~~only~~ one and ~~one~~ only one centre of gravity

CENTROID

This the point where the whole area of body is assumed to be concentrated.

Thus, when the thicness of the body is not consider then the centre of gravity and centroid of a body is same and passes through the same point. The plane figures like - rectangle, triangle, circle etc. have area only but no mass



cectriod wher the poi of $h/3$ & $b/2$

NOTE :-

(I) The axes in the above case

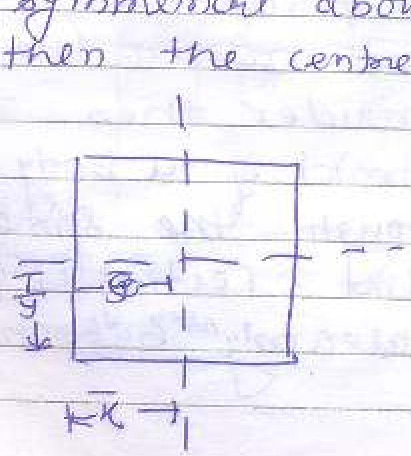
axes of Symmetry

Some times some figures are symmetrical about the X & y-axes, Say

if a body is symmetrical about x-axis then the centre of gravity will lie upon x-axis then only $\bar{y}$ have to be calculated.

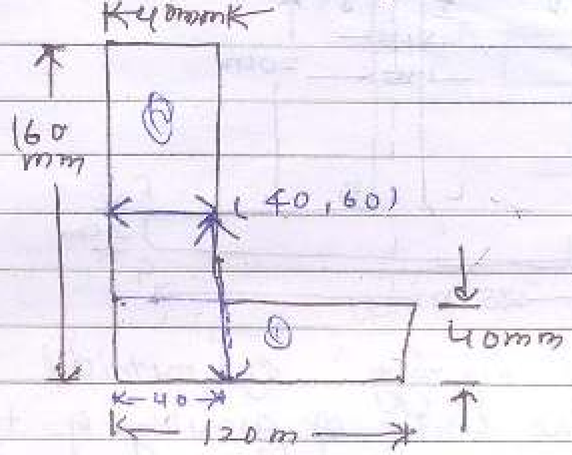
If a body is symmetrical about y-axis then the centre of gravity will lie upon y-axis then only $\bar{x}$ have to be calculated.

If a body symmetrical about both X-axis and y-axis then the centre of body is $(\bar{x}, \bar{y})$



Q find the position of centroid of L section

Q



Solⁿ $x_1 = \frac{40}{2} = 20mm$

$y_1 = \frac{160}{2} = 80mm$

$x_2 = 40 + \frac{80}{2} = 80mm$

$y_2 = \frac{40}{2} = 20mm$

$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2}$

$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2}$

(i) $a_1 = 40 \times 160 = 6400 mm$

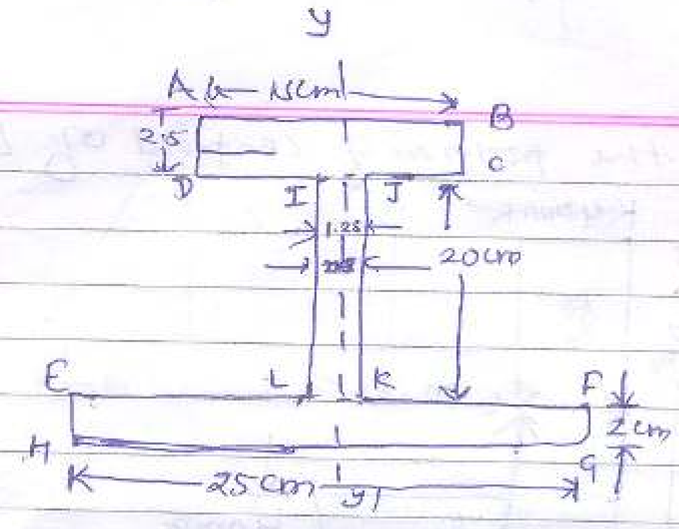
(ii) $a_2 = 80 \times 40 = 3200 mm$

$\bar{x} = \cancel{6400 \times 80 + 20 \times} \frac{3200 \times 20 + 6400 \times 80}{6400 + 3200}$

$= 40$

$\bar{y} = \frac{6400 \times 80 + 3200 \times 20}{6400 + 3200} = 60$

Q2 find the C.G. of I section with top ~~length~~ flange 15×2.5cm, web 20 × 2.5 cm and botom flange 25cm × 2cm



Ans. The section is ~~sy~~ symmetrical about YY' therefor. The centre of gravity of the section lie on yy' axis but the section is not symmetrical about x-axis.

Let the whole section is divided into three reactangles.

Let $\bar{y}$ be the distance of C.G. from base GH

(i) Rectangle (1)

$A_1 = 15 \times 2.5 = 37.5 \text{ cm}^2$

$y_1 = \frac{2.5}{2} + 20 + 2 = 23.25 \text{ cm}$

(ii) Rectangle (2)

$A_2 = 1.25 \times 20 = 25 \text{ cm}^2$

$y_2 = \frac{20}{2} + 2 = 12 \text{cm}$

(iii) Rectangle (3)

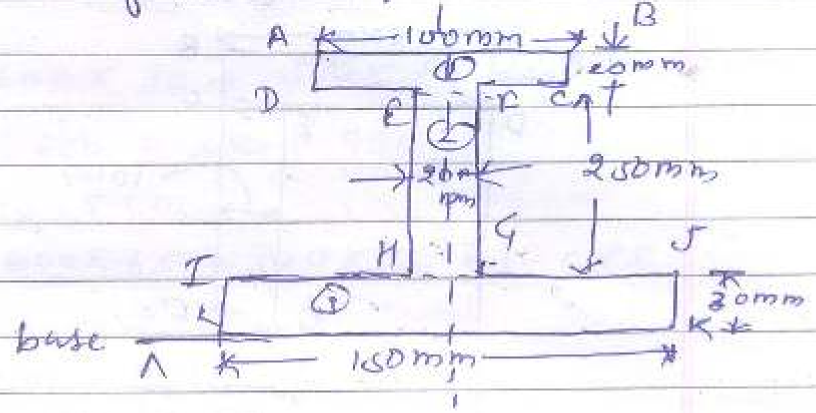
$A_3 = 2 \times 25 = 50 \text{ cm}^2$

$y_3 = \frac{2}{2} = 1 \text{ cm}$

$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{37.5 \times 23.25 + 25 \times 12 + 50 \times 1}{37.5 + 25 + 50}$

$= \frac{871.875 + 300 + 50}{112.5} = 10.86$

Q find the position of Centroid of I section



Ans

The section is symmetrical about yy' axis
The centre of gravity of the section is lie on yy'.
whole section divided into ③ rectangle

① Rectangle ①

$A_1 = 20 \times 100 = 2000 mm^2$

$y_1 = \frac{20}{2} + 250 + 30 = 290 mm$

② Rectangle ②

$A_2 = 20 \times 250 = 5000 mm^2$

$y_2 = \frac{250}{2} + 30 = 125 + 30 = 155 mm$

③ Rectangle ③

$A_3 = 30 \times 150 = 4500 mm^2$

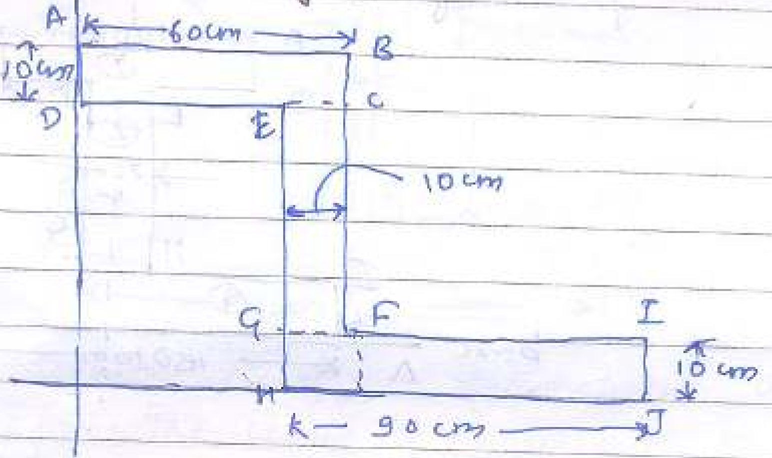
$y_3 = \frac{30}{2} = 15 mm$

$$y = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{2000 \times 290 + 5000 \times 155 + 4500 \times 15}{2000 + 5000 + 4500}$$

$$= \frac{580000 + 775000 + 67000}{11820} = 123.6956 mm$$

Q Find the C.G of Z-section as shown in figure



Let us divided the whole Section into (3) recatangles ABCD, CEFG, GHJI

Let the centre of gravity distance of the section is at a distance of $\bar{x}$ from the base AD

distance of c.g. from Base AD

(i) Area of ABCD $= 60 \times 10 = 600\,cm^2$

$A_1 = 600\,cm^2$.

$x_1 = \frac{60}{2} = 30\,cm$

$y_1 = \frac{10}{2} + 60 = 65\,cm$

(ii) Area of ECGF $A_2 = 50 \times 10 = 500\,cm^2$

$x_2 = \frac{10}{2} + 50 = 55\,cm$

$y_2 = \frac{50}{2} + 10 = 35\,cm$

(iii) Area of GHIJ $A_3 = 90 \times 10 = 900\,cm^2$

$x_3 = \frac{90}{2} + 50 = 95\,cm$

$y_3 = \frac{10}{2} = 5\,cm$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}, \quad \bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

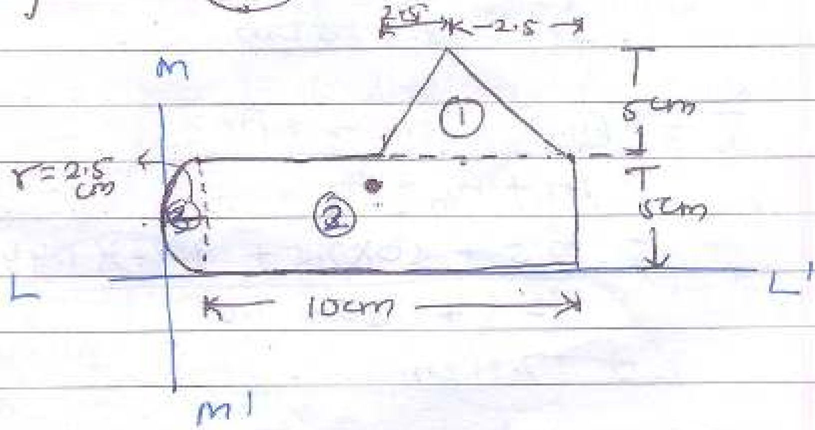
$$= \frac{600 \times 30 + 500 \times 55 + 900 \times 95}{600 + 500 + 900}$$

$$= 65.5 \text{ cm}$$

$$\bar{y} = \frac{600 \times 65 + 500 \times 35 + 900 \times 5}{2000}$$

$$= 30.5 \text{ cm}$$

Q ~~Using~~ Determine the C.G. of the plane uniform Lamina



Let the

~~S.No.~~	~~Name~~	~~$\bar{x}$ (cm)~~	~~$\bar{y}$ (cm)~~	~~Area (cm²)~~
~~1~~	~~Triangle~~	~~$\frac{5}{2}+5=7.5$~~	~~$\frac{5}{3}+5$~~	~~$\frac{1}{2}\times5\times5=12.5$ cm²~~
~~2.~~	~~Rectangle~~			

① for the Triangle

$$x_1 = 2.5 + 5 + 2.5 = 10 \text{ cm}$$

$$y_1 = \frac{5}{3} + 5 = 6.67 \text{ cm}$$

$$A_1 = \frac{1}{2} \times 5 \times 5 = 12.5 \text{ cm}^2$$

(2) For the Rectangle

$x_2 = 2.5 + \frac{10}{2} = 7.5 cm$

$y_2 = \frac{5}{2} = 2.5 cm$

$A_2 = 10 \times 5 = 50 cm^2$

(3) For the semi circle

$r = 2.5 cm$, $A_3 = \frac{1}{2}\pi r^2 = \frac{1}{2} \times 3.14 \times (2.5)^2 =$

$y_3 = 2.5 cm$

$x_3 = \cancel{9.8} - \cancel{\frac{4 \times 2.5}{3 \times 3.14}}$ $\quad 2.5 - 1.06$

$= \cancel{9.8} - \cancel{1.06}$ $\quad = 1.44 cm$

$x_3 = \cancel{8.738 cm}$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$= \frac{12.5 + 50 \times 7.5 + 9.81 \times 1.44}{12.5 + 50 + 9.81}$$

$= 7.11 cm$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{12.5 \times 6.67 + 50 \times 2.5 + 9.8 \times 2.5}{72.31}$$

$= 3.22 cm$

Centre of gravity of solids :-

for the calculating the centre of gravity of solids take volume insted of area ~~or calculating~~

Volume of some solids :-

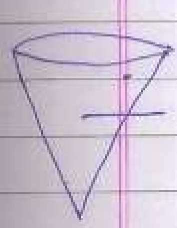
(i) Volume of sphere = $\frac{4}{3}\pi r^3$ { $r \to$ radius

(ii) volume of hemisphere = $\frac{2}{3}\pi r^3$

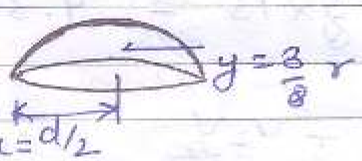
(iii) Volume of right circular cone = $\frac{1}{3}\pi r^2 h$

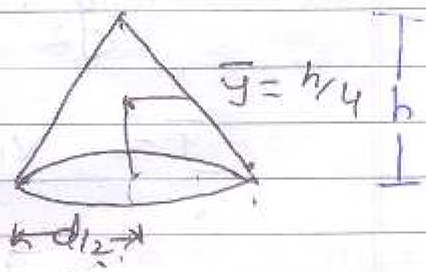
r = radius of right circular

h = height of cone.



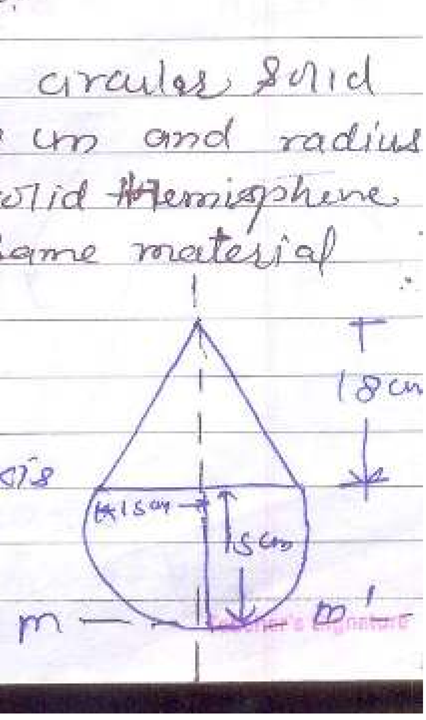
$\bar{x} = d/2$, $\bar{y} = h - \frac{h}{4} = \frac{3}{4}h$





Q. A Body consist of a right circular solid cone of height 18 cm and radius 15 cm placed on a solid hemisphere of radius 15 cm of the same material find the position of C.G.

Sol Since the composit body is symmetrical about y-axis so the C.G. will lie on y-axis and we shall



there fore find out $\bar{y}$ only.

Volume of Right circular cone

$$v_1 = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 15 \times 15 \times 18 = 1350\pi \, cm^3$$

Volume of Hemisphere

$$v_2 = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \pi \times 15 \times 15 \times 15$$

$$= 2250\pi \, cm^3$$

distance of c.g. from base MM'

$$y_1 = \frac{h}{4} + 15 = \frac{18}{4} + 15 = 19.5 \, cm$$

$$y_2 = 15 - \frac{3}{8} \times 15 = 9.38 \, cm$$

$$\bar{y} = \frac{v_1 y_1 + v_2 y_2}{v_1 + v_2}$$

$$= \frac{1350\pi \times 19.5 + 2250\pi \times 9.38}{1350\pi + 2250\pi}$$

$$= \frac{26325 + 21105}{3600} = \frac{47430}{3600}$$

$$\bar{y} = 13.175 \, cm$$

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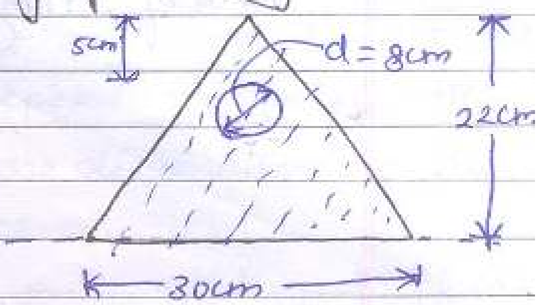
Q. find the centre of gravity of the given section

Sol :: This body symmetrical about y axis



Are of triangle

$A_1 = \frac{1}{2} \times 30 \times 22$

$= 330 \text{ cm}^2$

$y_1 = \frac{22}{3} = 7.33 \text{ cm}$

Area of circule

$A_2 = \pi \times 4 \times 4 = 50.26 \text{ cm}^2$

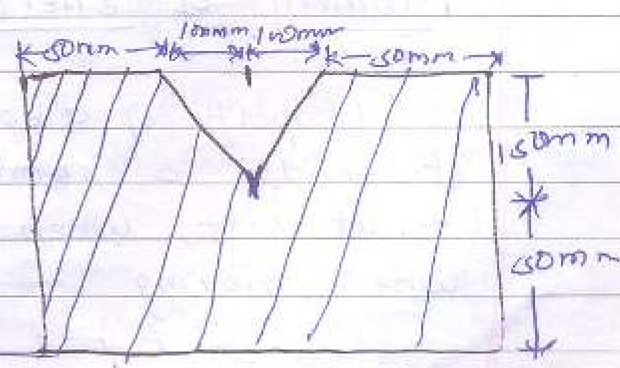
$y_2 = \frac{8}{2} + 9 = 13 \text{ cm}$

$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{330 \times 7.33 - 50.26 \times 13}{330 - 50.26}$

$= \frac{2418.9 - 653.38}{279.74} = \frac{1765.52}{279.74} = 6.31$

Q find the position of C.G. of a thin plate as shown in figure

Sol



This section is symmetrical about y-axis then only we find $\bar{y}$.

Now

area of reactangle $A_1 = 200 \times 300$

$= 60000 \text{ mm}^2$

$y_1 = 100 \text{ mm}$

Area of triangle $= \frac{1}{2} \times 200 \times 150$
$= 15000 \text{ mm}^2$

$y_2 = 150 - \frac{150}{3} + 50 = 150$
$= \frac{50}{150} = \cancel{86.6 \text{ mm}}\ 100$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 - A_2}$$

$$= \frac{60000 \times 100 - 15000 \times 100}{30000 - 5000}$$

$$= \frac{1500000 - 83000}{25000} = \frac{6 \times 10^9 - 15 \times 10^1}{25 \times 10^2}$$

$$= \cancel{56.68 \text{ mm}}\ \text{mm}$$

$$= \frac{4.5 \times 10^6}{25 \times 10^4} = \frac{4.5}{2.5} \times 10^2$$

$$= \frac{60000 \times 100 - 15000 \times 100}{60000 - 15000}$$

$$= \frac{3750}{45} = 83.33 \text{ mm}$$

Moment of Inertia

Inertia is the inherent property of a body by virtue of which it tends to remain in its own state i.e. at rest, when still, and in motion when moving.

A force is necessary to change its state and magnitude of the force depend upon the mass of the body. So the mass of a body is measure of it's inertia.

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~~Moment of Inertia~~ :-

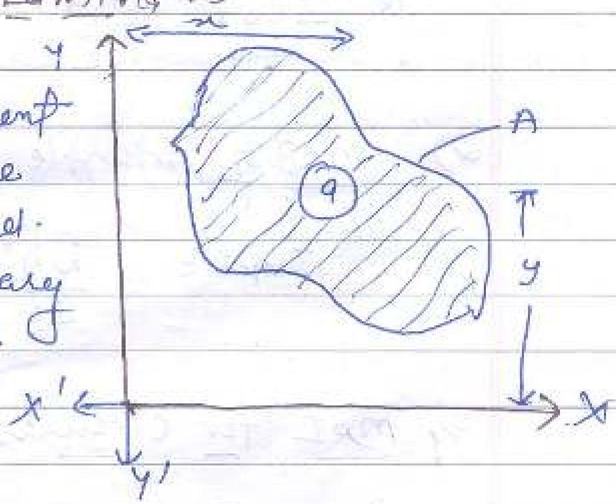
for a rotating body, about any axis, the measure of inertia not only depend on the quantity of its mass but also on the distribution of the mass from its axis of rotation.

Such a property of a rotating body is called rotational inertia or moment of inertia & generally denoted by 'I'.

NOTE :- moment of inertia is a measure of an object's resistance to changes in its rotation rate.

Moment of Inertia of Lamina :-

Consider a Lamina of a area A whose moment of inertia about some axis is to be determined.



Consider an elementary area 'a' which is the distance of x & y from yy' axis & xx' axis respectively.

The product of the area & the square of prependicular distance of its C.G. from the axis is M.O.I.

Since, moment of an area is again multiplied by the same distance, the moment of inertia is also called second moment of area. Now moment of area 'a' about X-axis

$I_{xx'} = ay^2$

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Moment of inertia of whole lamina of area 'A' about x-axis is

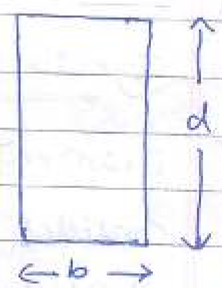
$$\sum I_{xx'} = \sum a y^2$$

Similarly, moment of inertia of whole area 'A' about YY' is

$$\boxed{\sum I_{yy'} = \sum a x^2}$$

Moment of Inertia of common standard section

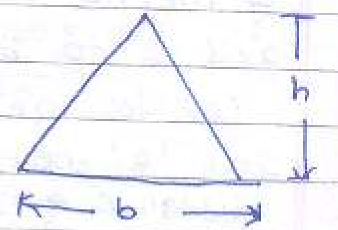
1) MoI of rectangle section



$$I_{xx'} = \frac{bd^3}{12}$$

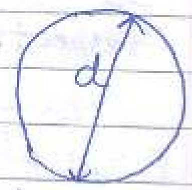
& $$I_{yy'} = \frac{db^3}{12}$$

2) MoI of triangle section



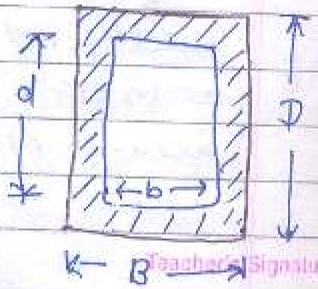
$$I_{xx'} = \frac{bh^3}{36}$$

3) MoI for circular section



$$I_{xx} = I_{yy} = \frac{\pi}{64} d^4$$

For hollow section (Reactangle)



$$I_{xx'} = \frac{BD^3}{12} - \frac{bd^3}{12}$$

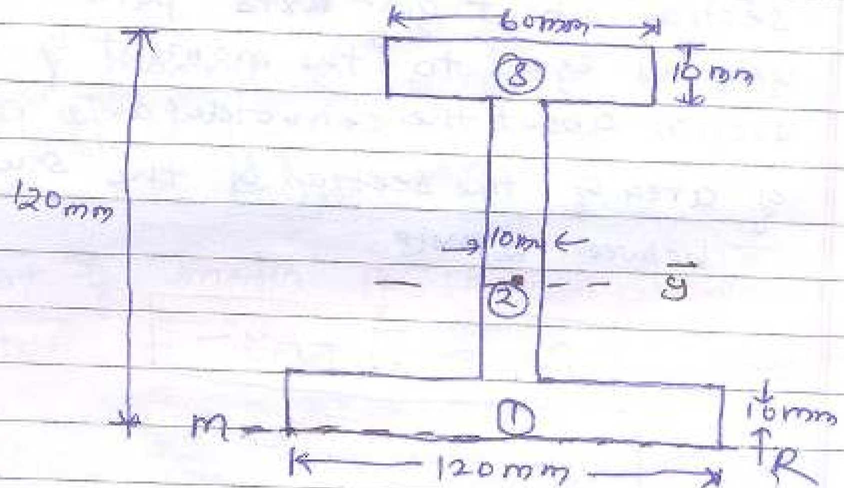
$$I_{yy'} = \frac{DB^3}{12} - \frac{db^3}{12}$$

Theorem of Parallel axis for finding M.O.I.

It states, "The moment of inertia of plane section about any axis parallel to the centroid axis is equal to the moment of inertia of the section about the centroidal axis pluse the product of area of the section & the square of distance between 2 axis

$$\boxed{I_{AB} = I_G + Ay^2}$$

Q Find the moment of inertia about the horizontal axis through the centroid of the section as shown in figure



Sol^n The ~~Symm~~ section has symmetrical through Y-axis. Dividing the section into three sub section. 1, 2, 3

Then for subsection (1) area of subsection

$a_1 = 120 \times 10 = 1200\ mm^2$

Distance of y_1 from base MR

$y_1 = \frac{10}{2} = 5mm$

for subsection (2)

area of subsection $= 100 \times 10 = 1000 mm^2$

Distance of y_2 from base MR

$y_2 = \frac{100}{2} + 10 = 60mm$

for subsection (3)

area of subsection $a_3 = 60 \times 10 = 600 mm^2$

distance of y_3 from base MR $= \frac{10}{2} + 110 = 115mm$

(4×4) = 16

distance of C.G. for whole section from MR base

$$C.G. = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$= \frac{1200 \times 5 + 1000 \times 60 + 600 \times 115}{1200 + 1000 + 600}$$

$$= \frac{6000 + 60000 + 69000}{2800}$$

$= 48.21\ mm$

This is distance of x-x' $(\bar{y})$ from base MR

Now MOI of subsection (1) about a line through G_1 parallel to MR

$$I_{G_1} = \frac{bd^3}{12} = \frac{120 \times 10^3}{12} = 10000\ mm^4$$

$$I_{MR_1} = I_{G_1} + a_1y^2 \quad (\text{when } y = \bar{y} - y_1)$$

$= 10000 + 1200(48.2 - 5)^2$

$= 2249488\ mm^4$

for subsection (2)

M.O.I. about x-x'

$$I_{G_2} = \frac{10 \times (100)^3}{12} = \frac{10^7}{12}$$

$$I_{MR_2} = I_{G_2} + a_2y^2 \quad \text{when } y = y_2 - \bar{y}$$

$$= \frac{10^7}{12} + 1000 \times (60 - 48.2)^2$$

$= 972573.33\ mm^4$

for subsection (3)

MOI $\quad I_{G_3} = \frac{60 \times 10^3}{12} = 5000\ mm^4$

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$I_{mR_3} = 5000 + 600 \times (115 - 48.2)$

$= \cancel{5000 + 600 \times 5.5}$

$\cancel{= 38000\ mm^4}$

$$\boxed{I_{mR_3} = 2682344\ mm^4}$$

The moment of intertia of whole section from xx'

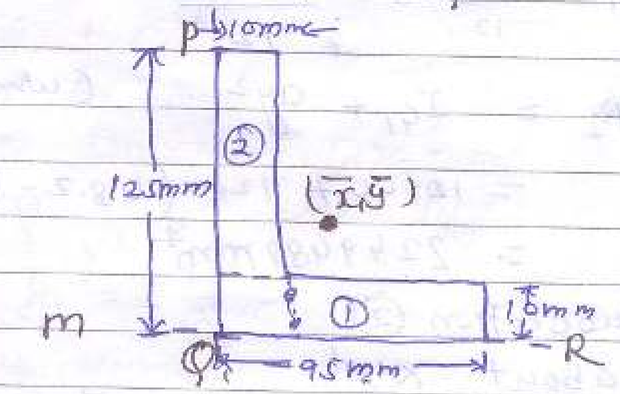
$I_{xx} = I_{mR_1} + I_{mR_2} + I_{mR_3}$

$= 2249480 + 972573 + 2682344$

$= 5904405\ mm^4$

$= 5.9 \times 10^6\ mm^4$

Q find the I_{xx} and I_{yy} for unequal section $125 \times 95 \times 10$ ~~mm~~



Sol^n Now cetre of gravity of whole section

The body is unsymmetrical so we find the $\bar{x}$ and $\bar{y}$ also

for ~~sub~~ this we divide the whole section into ② sub-section choose the base

Now sub-section ①

Area of subsection ①

$a_1 = 10 \times 95 = 950\ mm^2$

$x_1 = \frac{95}{2} = \cancel{8mm}\ 47.5$

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$y_1 = \frac{\cancel{115}}{\cancel{2}} + \frac{10}{2} = \cancel{67.5\ mm}\ 5mm$

for subsection ②

area $a_2 = 10 \times 115 = 1150\ mm^2$

$x_2 = \frac{10}{2} = 5mm$

$y_2 = \frac{115}{2} + 10 = 67.5\ mm$

Now C.G.

$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2} = \frac{950 \times 47.5 + 1150 \times 5}{950 + 1150}$

$= 24.22mm$

$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2} = \frac{950 \times 5 + 1150 \times 67.5}{950 + 1150}$

$= 39.22mm$

MOI of subsection ①

$I_{xx_1} = \frac{95 \times 10^3}{12} + \cancel{7916\ mm^4}\ 950 \times (34.22)^2 = \cancel{40425\ mm^4}\ 1120374\ mm^4$

$I_{yy_1} = \frac{10 \times 95^3}{12} + 950 \times (70.78) = 5473797\ mm^4$

MOI of substraction ②

$I_{xx_2} = \frac{10 \times 115^3}{12} + 950 \times (14.10)^2 = 1456265\ mm^4$

$I_{yy_2} = \frac{10 \times 115^3}{12} + 950 \times ($

$= 6722873$

$I_{xx} = 1120374 + 6722873 = 7843247\ mm^4$

$I_{yy} = 5473797 + 1456265 = 6930062\ mm^4$

$I_{zz} = 14773309\ mm^4$

moment of Interia about XX' for subsection

$I_{xx_1} = I_{G_1} + a_1 y^2 \qquad (\bar{y} - y_1)$

$I_{G_1} = \frac{bd^3}{12} = \frac{10 \times 115^3}{12} = 1267395.8 \text{ mm}^4$

$I_{xx_1} = 1267395 + 1150 \times (67.5 - 39.2)^2$

$= 2188418 \text{ mm}^4$

$I_{yy_1} = \frac{db^3}{12} + a_1 x^2$

$= \frac{115 \times 10^3}{12} + 1150 \times (24.2 - 5)^2$

$= 433519 \text{ mm}^4$

for subsection ②

$I_{xx_2} = I_{G_2} + a_2(\bar{y} - y_2)^2$

$= \frac{bd^3}{12} + a_2(\bar{y} - y_2)^2$

$= \frac{95 \times 10^3}{12} + 950 \times (39.2 - 5)^2$

$I_{xx_2} = 1119074 = 1.1 \times 10^6 \text{ mm}^4$

$I_{yy_2} = \frac{10 \times 95^3}{12} + 950(47.5 - 24.2)^2$

$= 1230224 \text{ mm}^4$

$I_{xx} = I_{xx_1} + I_{xx_2}$

$= 2188418 + 1119074 = 3307492 \text{ mm}^4$

$= 3.3 \times 10^6 \text{ mm}^4$

moment of Inertia of whole section of about yy1

$I_{yy} = I_{yy(1)} + I_{yy(2)}$
$= 433519 + 1230224$
$= 1663743 \ mm^4$
$= 1.6 \times 10^6 \ mm^4$

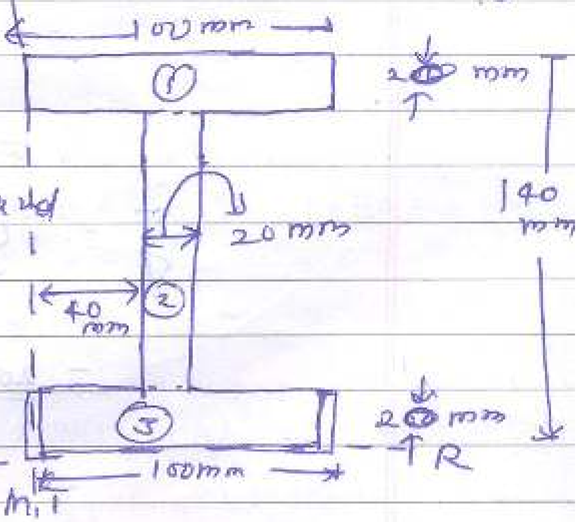
$I_{zz} = I_{xx} + I_{yy}$
$= (3.3 \times 10^6 + 1.6 \times 10^6) \ mm^4$
$= 4.9 \times 10^6 \ mm^4$

Q. find the moment of Inertia about the centrotial axis I_{xx} and I_{yy} of plane section as given

Sol^n

This section is symmetrical about axes so we can find both $\bar{x}$ and $\bar{y}$.

Divide this section into 3 three Reactgle parts



for ① subsection

area of subsection $a_1 = 100 \times 20 = 2000 \ mm^2$

$y_1 = \frac{20}{2} + 120 = 130 mm$

$x_1 = \frac{100}{2} = 50 \ mm$

for subsection ②

area $a_2 = 20 \times 100 = 2000 \ mm^2$

$y_2 = \frac{100}{2} + 20 = 70 mm$

$x_2 = \frac{20}{2} + 40 = 50 mm$

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For subsection ③

area of subsection $a_3 = 20 \times 100 = 2000\ mm^2$

$x_3 = \frac{100}{2} = 50mm$

$y_3 = \frac{20}{2} = 10mm$

C.G. $\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$

$= \frac{2000 \times 50 + 2000 \times 50 + 2000 \times 50}{2000 + 2000 + 2000}$

$= \frac{100000 + 100000 + 100000}{6000}$

$= \frac{\overset{50}{\cancel{300000}}}{\cancel{6000}}$

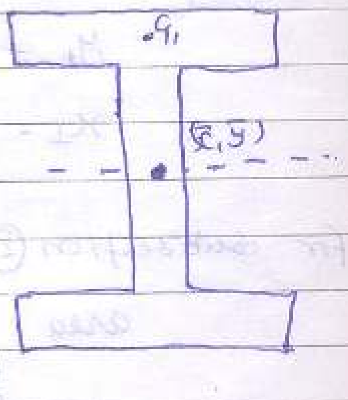
$= 50\ mm$

$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$

$= \frac{2000 \times 130 + 2000 \times 70 + 2000 \times 10}{2000 + 2000 + 2000}$

$= \frac{\cancel{2000}[130 + 70 + 10]}{\cancel{2000}(3)} = \frac{210}{3} = 70mm$

Now C.G. is



Now MOI of subsection ①

$$I_{xx'_1} = I_{G_1} + a_1 y^2$$

$$I_{G_1} = \frac{bd^3}{12} = \frac{100 \times 20^3}{12} = 66666\ mm^4$$

M.O.I. of subsection ①

$$I_{xx'_{①}} = 66666 + 2000 \times (60)^2 = 7266666\ mm^4$$

$$I_{yy'_{①}} = \frac{db^3}{12} + a_1 x^2$$

$$= \frac{20 \times 100^3}{12} + 2000 \times (0-0)^2 = 1666666\ mm^4$$

MOI for subsection - ②

$$I_{xx'_{②}} = \frac{bd^3}{12} + a_2 y^2$$

$$= \frac{20 \times 100^3}{12} + 2000\,(0-0)^2 = 1666666\ mm^4$$

$$I_{yy'_{②}} = \frac{db^3}{12} + a_2 x^2$$

$$= \frac{100 \times 20^3}{12} + 2000\,(0-0)^2 = 66666\ mm^4$$

MOI for subsection - ③

$$I_{xx^{①}_{③}} = \frac{bd^3}{12} + a_3 y^2$$

$$= \frac{100 \times 20^3}{12} + 2000 \times (60)^2$$

$$= 66666 + 7200000$$

$$= 7266666\ mm^4$$

$$I_{yy^{①}_{3}} = \frac{db^3}{12} + a_3 x^2$$

$$= \frac{20 \times 100^3}{12} + 2000\,(0)^2 = 1666666\ mm^4$$

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$$I_{xx'} = I_{xx'_1} + I_{xx'_2} + I_{xx'_3}$$

$$= \cancel{9659998\,mm^4} \quad 16199998\ mm^4$$

$$= 9.61 \times 10^6\ mm^4 \quad = 1.6 \times 10^7\ mm^4$$

$$I_{yy'} = I_{yy'_1} + I_{yy'_2} + I_{yy'_3}$$

$$= 3399998\ mm^4$$

$$= 3.3 \times 10^6\ mm^4$$

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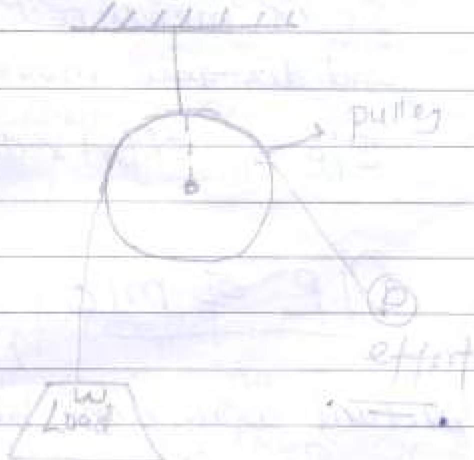
MACHINE :-

A machine is a device whic receive energy in same available form and uses it for doing a perticular work.

ex - The steam engine convert the energy provided by steam into the motion of transition

Lifting m/c :-

A lifting m/c is a device to overcome a force or load (W) applied at one point by means of another force called effort (P), applied at another point



NOTE :- A m/c doesn't work by itself, only when energy is supply to it.

Mechanical Advantages (M.A.) :-

The ratio of load lifted (W) to the applied effort

$$M.A. = \frac{\text{Load Lifted}}{\text{effort applied}} \frac{(W)}{(P)}$$

$$\boxed{M.A. = \frac{W}{P}}$$

NOTE :- The effort applied is generaly smaller then the load lifted.

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So that M.A. is grates then 1

(V.R.) Velocity Ratio:-

It is the ratio of the distance (y) moved by the effort to the distance (x) moved by the load.

mathemetically

$$V.R. = \frac{\text{distance (y) moved by the effort}}{\text{distance (x) moved by the load}}$$

$$\boxed{V.R. = \frac{y}{x}}$$

Input of M/c :-

Input of a m/c is work done on the m/c

It is measured by the product of effort and distance moved by effort

I/P = effort x distance moved by effort

O/P of M/c

The output of a m/c is the actual work done by the m/c.

It is ~~mea~~ measured by the product of load to the distance moved by the load.

O/p of m/c = load x distance moved by load.

$$\boxed{O/_P = W \times x}$$

Efficiency of M/c :-

It is the ratio of output of m/c to the Input of m/c,

It is generally expressed by precentage

And it is denoted of η

$$\boxed{\eta = \frac{O/p}{I/p}} \qquad \eta = \frac{m.A}{v.R}$$

Ideal m/c :-

If the effeciency of m/c is 100% that is output of m/c equal to the input of m/c is called Ideal m/c.

Simple m/c :-

A simple m/c may be defined as a device, which enable us to do some useful work at some point ~~o~~ to overcome some Resistance, when an effort is applied to it at same another ~~convenient~~. Convenient.

Compound machine :-

A compound machine may be defined as a device consisting a no of simple m/c which enable us to do some work at a faster speed with a much less effort as compare to simple m/c.

18/4/16

Relation between Efficiency (η), Velocity Ratio (V.R.) & M

Let W = load lifted by machine
P = effort applied
y = distance moved by effort
x = distance moved by load.

Then $\boxed{M.A. = \frac{W}{P}}$

And velocity ratio

$$\boxed{V.R. = \frac{y}{x}}$$

Input of a machine $= Py$
O/p $= Wx$
then

$$\eta = \frac{O/p}{I/p} = \frac{Wx}{Py}$$

$$\boxed{\eta = \frac{MA}{V.R.}}$$

friction in Machine :-

There are always some loses in every machine and mostly they are due to friction. The machine friction may be expressed either from the effort side or the load side.

If expressed on the effort side, the friction may be defined as an additional effort required to overcome

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the frictional force.

Let P = actual effort (considering the m/c friction) required to lift a load.

W = actual load, (considering the m/c friction) lifted overcome.

P_i = ideal effort (considering the frictionless) required to ~~lifted~~ lift a load.

W_i = ideal load (considering the frictionless) lifted overcome.

f_P = loss of effort due to friction

& f_W = loss of load due " ".

then $\boxed{f_P = P - P_i}$ — ①

$\boxed{f_W = W_i - W}$ — ②

to calculate P_i consider an ideal m/c in which $\eta = 1$

we know that

$$M.A. = \eta \times V.R.$$

$$M.A. = 1 \times V.R. \quad (\text{for ideal})$$

$$\boxed{\frac{W}{P_i} = V.R.} \text{ — ③} \quad \boxed{P_i = \frac{W}{V.R.}} \text{ — ③}$$

substituting the value of P_i in ①

$$\boxed{f_P = P - \frac{W}{V.R.}} \text{ — ④}$$

calculate W_i again consider ideal m/c

$$M.A. = \eta \times V.R.$$

$$\frac{W_i}{P} = 1 \times V.R.$$

$$\boxed{W_i = P \times V.R.} \text{ — ⑤}$$

Substituting the ⑤ eqn in ② we have

$$\boxed{f_w = P \times V.R - W} \quad -⑥$$

Q A m/c Rise a load of 360N and through a distance of 200mm. The effort of a force 60N move by 1.8m during the process the calculate M.A., V.R., η, F_P, f_w.

Sol As per Given data

$W = 360\,N$, $P = 60\,N$

$x = .2m$, $y = 1.8m$

$$M.A. = \frac{W}{P} = \frac{360}{60} = 6$$

$$V.R. = \frac{y}{x} = \frac{1.8}{.2} = 9$$

$$\eta = \frac{M.A.}{V.R} = \frac{6}{9} = \frac{2}{3}$$

$$f_w = 60 \times 9 - 360$$
$$= 540 - 360$$
$$= 180\,N$$

$$f_P = P - \frac{W}{V.R.}$$
$$= 60 - \frac{360}{9}$$
$$= 60 - 40 = 20\,N$$

Q. 20% of the effort is lost in a lifting m/c when effort of 2000N is applied. the Velocity ratio of a machine is 20. Calculate the load lifted by m/c

Sol. Given $V.R. = 20$

$P = 2000 N$

$f_p = 20\% \text{ of } P$

$= 2000 \times \frac{20}{100} = 400 N$

$f_p = 400 N$

$\because F_p = P - P_i$

$P_i = 1600 N$

$\because P_i = \frac{W}{V.R.}$

$W = P_i \times V.R.$

$= 1600 \times 20$

$W = 32000 N$

$M.A. = \frac{W}{P} = \frac{32000}{2000} = 16$

$\eta = \frac{M.A.}{V.R.}$

$= \frac{16}{20} = 0.8$

$\boxed{\eta\% = 80\%}$

Q A m/c required an effort 5N to lift a load 60N. The velocity Ratio of a machine is 20 find the loss of load due to friction and efficiency of m/c

Sol

Given load $w = 60N$

$P = 5N$

$M.A. = \frac{w}{P} = 12$

$\because V.R. = 20$

$\because \eta = \frac{M.A.}{V.R.} = \frac{12}{20} = .6$

$= 60\%$

$f_w = P \times V.R - w$

$= 5 \times 20 - 60$

$= 40N$

Q In a certain machine A load of 100N is lifted by an effort 8N and efficiency of 60% find the effort loss and load loss due to

$M.A = \frac{w}{P} = \frac{100}{8} = 12.5$

$\therefore \eta = \frac{M.A}{V.R.}$

$V.R = \frac{M.A}{.6} = \frac{12.5}{.6} = 20.83$

$f_p = P - P_i$

$\because P_i = \frac{w}{V.R.} = \frac{100}{20.83} = 4.8N$

$f_p = 8 - 4.8 = 3.2N$

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$fw = W_i - w$

$\because W_i = P \times V.R.$

$= 8 \times 20.83$

$= 166.64$

$fw = 166.64 - 100$

$= 66.64 N$

Q If a lifting m/c the effort has to move through 1m in order to lift the load through 10 mm If the efficiency of m/c is 60%. Determine the load that can be lifted by an effort of 25 N

Sol

Given $y = 1m$

$x = 10mm = 10 \times 10^{-3} m$

$P = 25N$

$\eta = 0.6$

$\therefore V.R. = \frac{y}{x}$

$= \frac{1}{10 \times 10^{-3}} = \frac{10^3}{10} = 100$

$\therefore \eta = \frac{M.A.}{V.R.}$

$M.A. = 0.6 \times 100$

$= 60$

$W = P \times M.A.$

$= 25 \times 60$

$= 1500 N$

* Reversibility of m/c:-

when the m/c is capabl[e of] doing work in the reverse direction due t[o] its load, as the effort is totally zero (re[moved]) the m/c said to be reversible and the action of such a m/c is known as Reversibility of machine.

Let W = load lifted by m/c
P = effort required for lifing
y = distance moved by effort
x = distance moved by load

so that

$$M.A. = \frac{W}{P}$$

output of m/c = $W \times x$

I/P = $P \times y$

friction losses in machine = I/P − O/P
= $P \times y - W \times x$

in a reversible m/c the output of m/c should be more then the m/c friction when effort is zero. i.e.

$$W \times x > (P \times y - W \times x)$$

$$2Wx > Py$$

$$\frac{Wx}{Py} > \frac{1}{2}$$

$$\frac{W/P}{y/x} > \frac{1}{2}$$

$$\frac{MA}{VR} > \frac{1}{2}$$

$\boxed{\eta > \frac{1}{2}}$ it means $\boxed{\eta > 50\%}$

Hence, for a m/c to be reversible the efficiency of the m/c is to be grater than 50%.

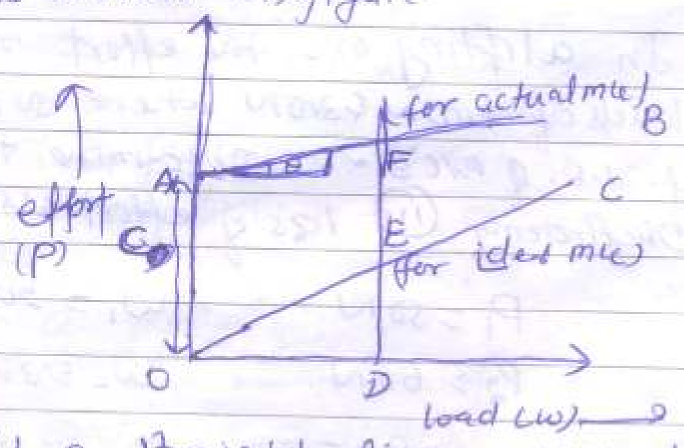
~~Law~~ of machine

Self locking m/c If a machine is not capable to work in reverse direction when effort is zero.

Law of Machine :-

- The relationship between load lifted by machine when effort applied in a machine is called Law of machine. ~~This is~~

This is found by conducting experimently in which effort required for a lifting different loads and the load verses effort graph is denoted as shown in figure.



This is a straight line which doesn't passes through the origin. The law of m/c is to be expressed

$$\boxed{P = mW + C}$$

where P = effort, W = load, m = slope = $\tan\theta$ & C = intercept.

If the machine is ideal the $\eta = 1$

$M.A. = \eta \times V.R.$

$$\boxed{\frac{W}{P} = V.R.}$$

This is a straight line (OC) and passing through the origin.

The vertical line DEF corresponding to given load OD and effort as shown in figure then

$DE = P_i$ (ideal effort)

$DF = P$ (actual effort)

Then loss of friction $= P - P_i$

$$\boxed{\eta = \frac{P_i}{P} = \frac{DE}{DF}}$$

Q In a lifting m/c the effort required to lift loads of 200N & 300N where 50N & 60N respectively. If V.R. of m/c is 20. Determine the (i) law of m/c (ii) efficiency (iii) loss of effort also both.

$P_1 = 50N$ | $W_1 = 200N$

$P_2 = 60N$ | $W_2 = 300N$

$V.R. = 20$

for (1)

$50 = 200m + c$ — (1)

for (2)

$60 = 300m + c$ — (2)

(2) − (1)

$300m + c - 200m - c = 60 - 50$

$100m = 10$

$$\boxed{m = \frac{1}{10}} \qquad \boxed{c = 30}$$

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$\frac{p x y}{}$ — $\frac{M.A.}{V.R}$

(i) efficiency of both load

$M.A._1 = \frac{W_1}{P_1}$, $MA_2 = \frac{W_2}{P_2}$

$= \frac{200}{50}$ $MA_2 = \frac{300}{60}$

$= 4$ $= 5$

$\eta_1 = M.A \cancel{\times} V.R.$ $\eta_2 = 5 \cancel{\times} 20$

$= 4 \cancel{\times} 20$ $= 0.25$

$= 0.2$

$\boxed{\eta_1 \% = 20\%}$ $\boxed{\eta_2 \% = 25\%}$

(ii) loss of effort in both

$f_{P_1} = P_1 - \frac{W_1}{V.R}$ $f_{P_2} = P_2 - \frac{W_2}{V.R}$

$= 50 - \frac{200}{20}$ $= 60 - \frac{300}{20}$

$= 40 N$ $= 45 N$

Pulley:- A pulley is essential ~~or~~ a metallic or wooden wheel which is capable to rotate about an axis

System of pulley:- A no. of pullies are so arranged that the composit system result into a gain in mechanical adiantages.

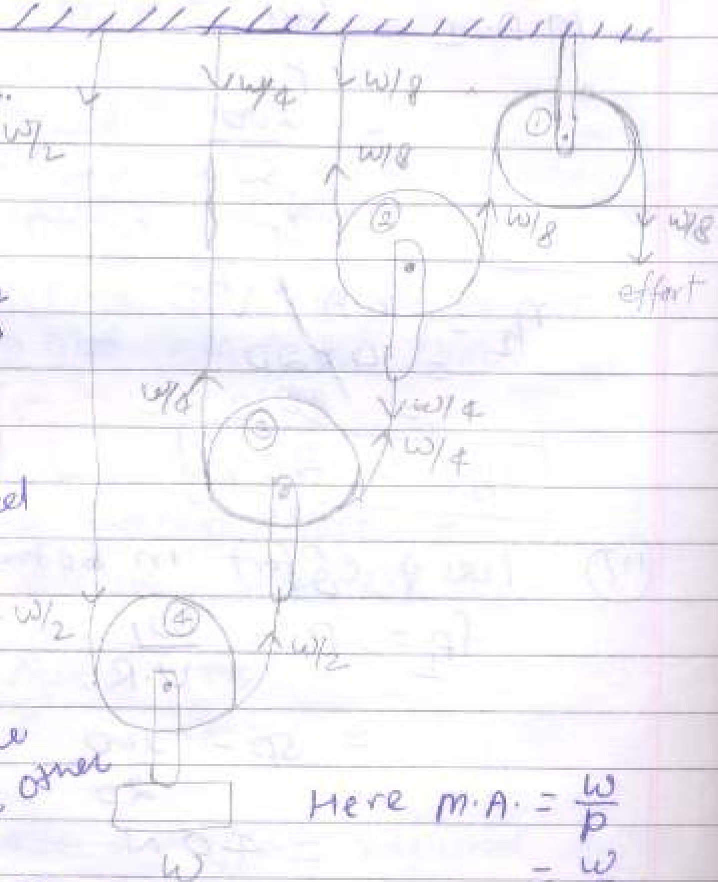
Depending upon their arrangement the pullies are classified into the three system-

(a) first order pully system.

(b) Second order pulley system.

(c) Third order pulley system

(a) first order pulley system :-



In this system first pulley is fix at the top support. Then one end of a rope fix at the fix support is made half term at pulley ~~two~~ (2) And passed over the pulley first. The effort is applied at the free end. pulley (3) is suspended By the Rope whose one end is fix at the fix support and the other end ~~to~~ pulley (2) similarly also pulley (4)

Here $M.A. = \frac{W}{P}$

$= \frac{W}{W/8}$

$\boxed{M.A. = 8 = 2^3}$

consider the machine is ideal then $\eta = 1$

$\therefore M.A. = \eta \times V.R$

$\boxed{M.A. = V.R. = 2^3}$

This V.R. is obtained only 3 movable pulley

If we take n pulleies then $\boxed{V.R. = 2^n}$

gmf

Q) What force effort is required to rise a load $W = 12,500\,N$ and efficience is 84%. for first order pulley system

Sol^n

As per given data :: $V.R. = 2^3 = 8$

$M.A = \eta \times V.R.$

$= .84 \times 8$

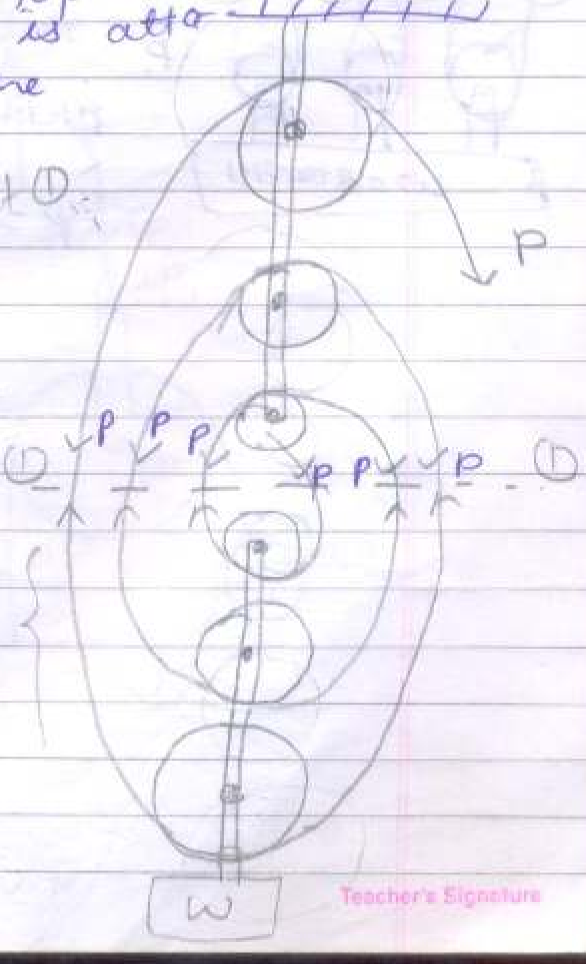
$= 6.72$

$\because M.A. = \frac{W}{P} = \frac{12,500}{P} = 6.72$

$P = \frac{12500}{6.72} = 1860.11\,N$

(ii) Second order pulley system

we prepare two block of pulley. the first one is fix with fixed support at the top. One end of a Rope is attached at the hook at the botom of the upper block. Then the rope passes round all the pulleies as shown in figure. The effort P is apply at the other end of the rope, and the load is attach at the botom hook of lower block.



Total $P+P+P+P+P+P = W = 6P$

$M.A. = \frac{W}{P} = \frac{6P}{P} = 6$

$M.A. = 6 = 2 \times 3$

Consider Ideal m/c $\eta = 1$

$\boxed{M.A. = V.R = 2 \times 3}$ This value of V.R is for three movable pulley If ~~two~~ we

Consider the n pulley in the system then

$\boxed{M.A. = V.R = 2n}$

Q. Calculate the effort for a 20000N load lifted by Given pulley system the efficiency of the system is to be 70%.

$W = 20000\,N$

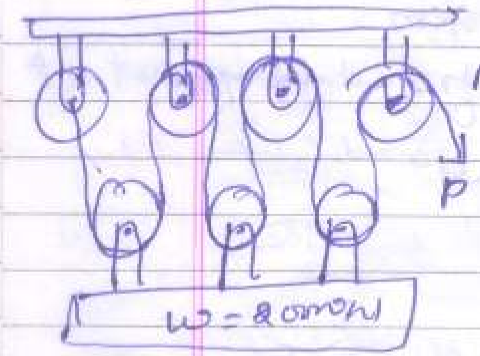
Here movable ~~two~~ pulley is three

$\eta = .7$

for this pulley system

$V.R = 2 \times 3$

$= 6$



$M.A. = \eta \times V.R.$

$= .7 \times 6$

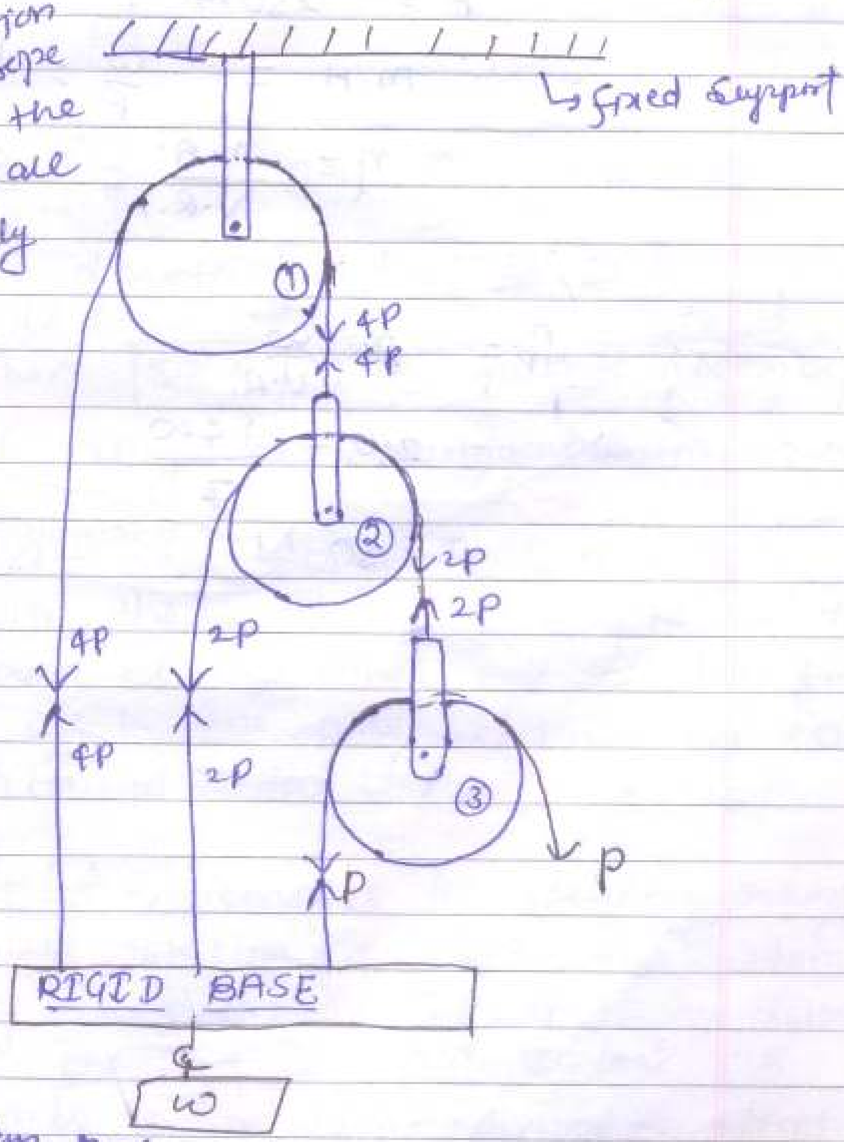
$= 4.2$

$M.A = \frac{W}{P}$

$P = \frac{W}{M.A.} = \frac{20000}{4.2} = 4761.9\,N$

Third order pulley system

In the third order pulley system the first pulley is fixed at the top fix support and a rope passes over this pulley whose one end is attached to a rigid base and the another end to pulley no. 2 another rope passes over this second pulley and its one end is fix at the rigid base while the other end to pulley no 3



Neglating the friction the tension in the rope which passes over the pulley 1, 2 & 3rd are 4P, 2P & P respectively.

A total effort is $P + 2P + 4P = 7P$ is acting at the rigid base to lift the load. so that

$\boxed{W = 7P}$

& $\boxed{P = \frac{W}{7}}$

$\because M.A. = \frac{W}{P} = \frac{W}{W/7}$

$\boxed{M.A. = 7}$

for an Ideal system $\eta = 1$

$M.A. = V.R. = 7$

$V.R. = 2^3 - 1$

If we have n pulley system then

$$\boxed{V.R. = 2^n - 1}$$

Q. In a lifting m/c the pulley are arranged in the third order system with three pulley. A load of 1400N is lifted by an effort of 250 N find the η & fp ?

Given In third order pulley stems

$V.R. = 2^n - 1$ here $n = 3$ then

$V.R. = 2^3 - 1 = 7$; $W = 1400\,N$

$P = 250\,N$

$\because M.A. = \frac{W}{P} = \frac{1400}{250} = 5.6$

$\because \eta = \frac{M.A.}{V.R.} = \frac{5.6}{7} = 0.8$

$= 80\%$

$fp = P - \frac{W}{V.R.}$

$= 250 - \frac{1400}{7}$

$= 50\,N$

$V.R. = 2^3 - 1$

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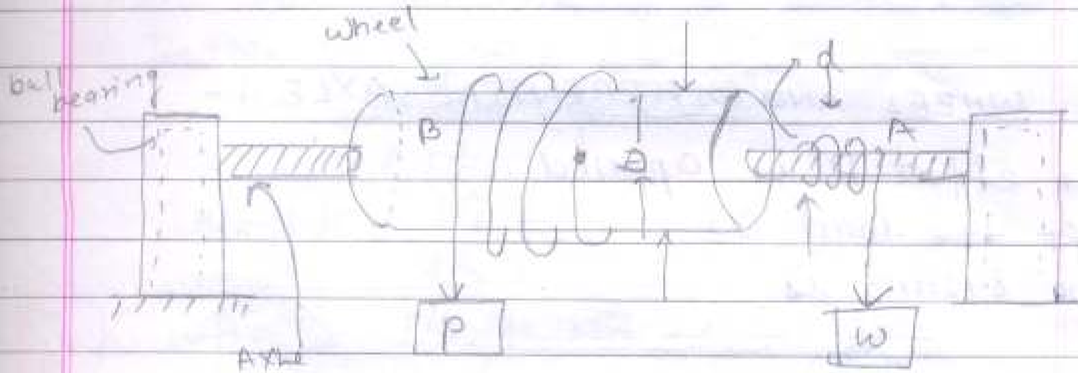
WHEEL AND AXLE :-

A wheel and axle a lifting m/c as shown in figure



It consists of an axle A attach to wheel B.

Let the diameter of wheel is D and Diameter of of axle is d.

Then $\boxed{D > d}$ The complete assembly is mounted using ball bearing on an fix support

Rope arrangement :- one end of a rope is attach with the wheel using a pin and then wound around it the other end is left free to apply effort. The other end of of this axle rope is attached to the load

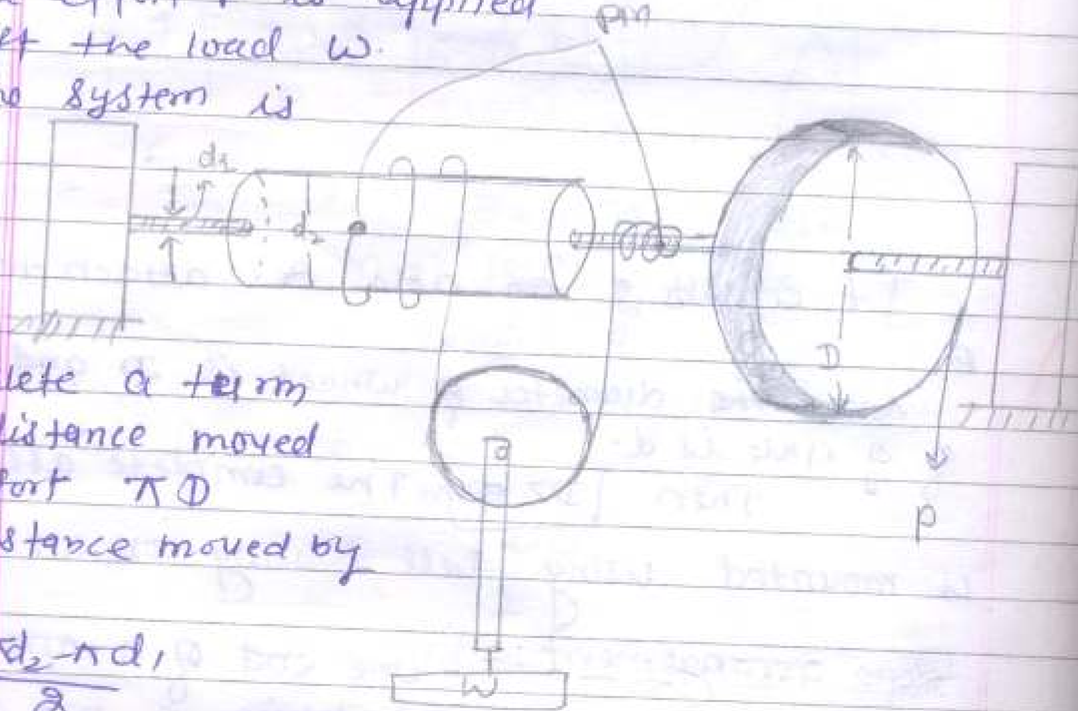
Velocity Ratio :- consider the assembly taking one complete rotation so that the distance moved by the effort is πD and the distance moved by the load is πd so that V.R. is given by = $\frac{\text{distance moved by effort}}{\text{'' '' '' load}}$

$$V.R. = \frac{\pi D}{\pi d}$$

$$\boxed{V.R. = \frac{D}{d}}$$

WHEEL AND DIFFERENTIAL AXLE :-

Let the effort P is applied to lift the load W. and the system is



complete a term the distance moved by effort πD and distance moved by load

$$= \frac{\pi d_2 - \pi d_1}{2}$$

$$= \frac{\pi}{2}(d_2 - d_1)$$

So the velocity Ratio is given by

$$V.R. = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

$$= \frac{\pi D}{\pi (d_2 - d_1)/2} = \frac{2D}{d_2 - d_1}$$

$$\boxed{V.R. = \frac{2D}{d_2 - d_1}}$$

$I = AK^2$
$K = \sqrt{\frac{I}{A}}$

Radius of Gyration (K) -

mass or area may be concentrated at a point. The distance of this point from the axis of rotation is called Radium of Gyration (K.)

The moment of inertia may be written

$I = AK^2$

Then $\boxed{K = \sqrt{\frac{I}{A}}}$

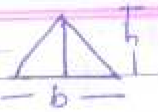
where I - Inertia, A = Area of section.

A[1]

23/04/16

Summary

Centroid of triangle



$\bar{x} = b/2 \quad \bar{y} = h/3$

for n no of subsection

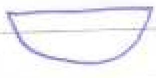
$$\bar{x} = \frac{a_1x_1 + a_2x_2 + \dots + a_nx_n}{a_1 + a_2 + \dots + a_n}$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + \dots + a_ny_n}{a_1 + a_2 + \dots + a_n}$$

If the system is symmetrical to axes then $\bar{x}$ & $\bar{y}$ is determind and if " about a single axis then according to the axis $\bar{x}$ or $\bar{y}$ is determind.

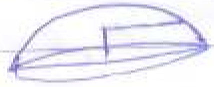
semicircle

$\bar{x} = d/2 \quad , \bar{y} = \frac{4r}{3\pi}$

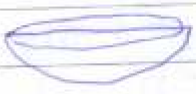


$\bar{x} = d/2 \quad , \bar{y} = r - \frac{4r}{3\pi}$

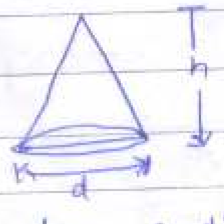
Hemisphere



$\bar{x} = d/2 , \quad \bar{y} = \frac{3r}{8}$



$\bar{x} = d/2 , \quad \bar{y} = r - \frac{3}{8}r = \frac{5}{8}r$



$\bar{x} = d/2 , \quad \bar{y} = h/4$

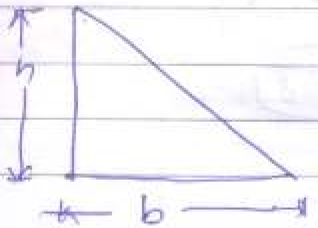
If two section is ~~into~~ overlaped or one empty section is in the another section then $\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2}$, $\bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2}$

$\bar{x} = b/2$, $\bar{y} = h/2$

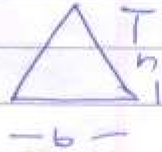
m o I

$I_{xx'} = \frac{bd^3}{12}$

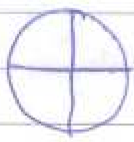
$I_{yy'} = \frac{db^3}{12}$



$\bar{x} = b/3$, $\bar{y} = h/3$



$I_{xx'} = \frac{bh^3}{36}$



$x = \bar{y} = d/2$, $I_{xx'} = I_{yy'} = \frac{\pi}{64} d^4$

for solids we find out the volume

$x = \frac{x_1 V_1 + x_2 V_2}{V_1 + V_2}$, $\bar{y} = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2}$

moI.

$I_{xx'} = I_G + Ay^2 = \frac{bd^3}{12} + A(\bar{y} - y_n)^2$

$I_{yy'} = I_G + Ax^2 = \frac{db^3}{12} + A(\bar{x} - x_n)^2$

$I_{zz'} = I_{xx'} + I_{yy'}$

$M.A. = \frac{W}{P}$, $V.R. = \frac{y}{x}$

$\eta = \frac{M.A.}{V.R.}$, $I/p = P \times y$

$O/p = W \times x$

$f_P = P - P_i$, $P_i = \frac{W}{V.R.}$

$f_W = W_i - W$, $W_i = P \times V.R.$

$\boxed{f_P = P - \frac{W}{V.R.}}$ $\boxed{f_W = P \times V.R. - W}$

for reversible m/c. $\boxed{\eta > 50\%}$

$\boxed{\text{Law of m/c } P = mW + C}$

$\boxed{\eta = \frac{P_i}{P}}$

$\boxed{M.A. = \eta \times V.R.}$

system of pulley

(a) for first order pulley system

$\boxed{\frac{w}{8} = P}$ $\boxed{\frac{w}{P} = 8}$

$\boxed{M.A. = 8}$

$M.A. = \eta \times V.R.$

for ideal m/c $\boxed{M.A. = V.R. = 2^3}$

$\boxed{V.R. = 2^3}$

If we take n pulleys then

$\boxed{V.R. = 2^n}$ when n = movable pulley and one is fixed at fix support.

$V.R. = 2^{n-1}$ where n total pulleys

(b) for second order pulley system

$\frac{w}{6} = P \Rightarrow \frac{w}{P} = 6 \Rightarrow \boxed{M.A. = 6}$

for ideal $M.A. = V.R. = 2 \times 3$

$\boxed{V.R. = 2 \times n}$ If n movable pulleys

$V.R. = 2 \times (n-1)$ n total pulleys

(c) Third order pulley system

$P = \frac{w}{7} \Rightarrow \frac{w}{P} = 7 \Rightarrow \boxed{M.A. = 7}$

$\boxed{V.R. = 2^n - 1}$, n = total no of pulleys

wheel and axel $\boxed{V.R. = \frac{D}{d}}$

wheel and differential axel

$$\boxed{V.R. = \frac{2D}{d_2 - d_1}}$$