

CENTROID OF PLANE FIGURES

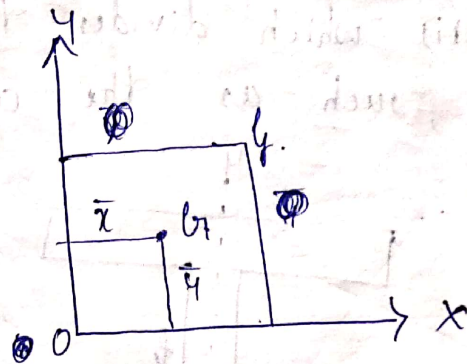
Centre of gravity:

It is the point where the whole weight of the body is assumed to be concentrated.

Centroid:

Centroid is the point where the whole area of the plane figure is assumed to be concentrated.

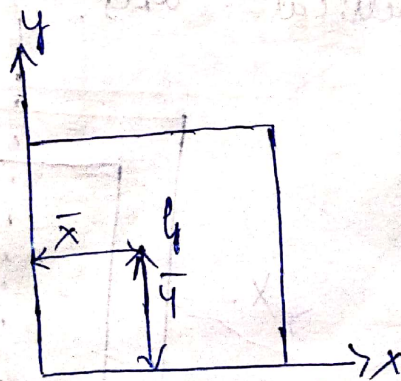
Centroid means the determination of $\bar{x}$ & $\bar{y}$ as shown in fig.



The difference between centre of gravity & centroid is the term centre of gravity applies to bodies with mass and weight, and centroid applies to plane areas.

Axes of reference:

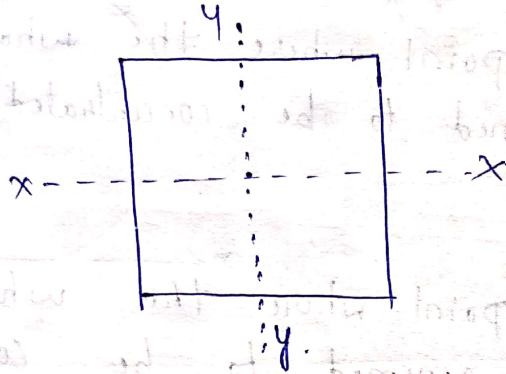
These



These are the axes with respect to which the centroid of a given fig. is determined.

Centroidal Axis:-

The axis which passes through the centroid of the given fig. is known as centroidal axis, such as the axis $x-x$ and the axis $y-y$ shown in fig.



Symmetrical Axis:-

It is the axis which divides the whole figure into equal parts, such as the axis $x-x$ and the axis $y-y$.

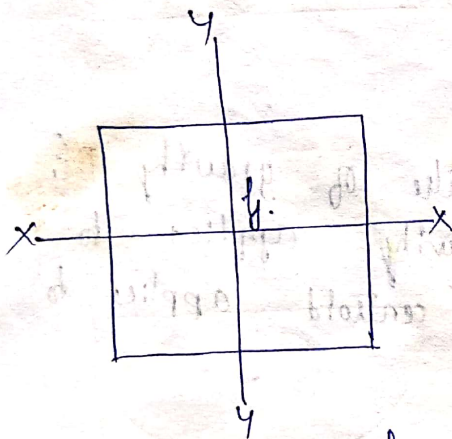


Fig (a)

Symmetrical axis.

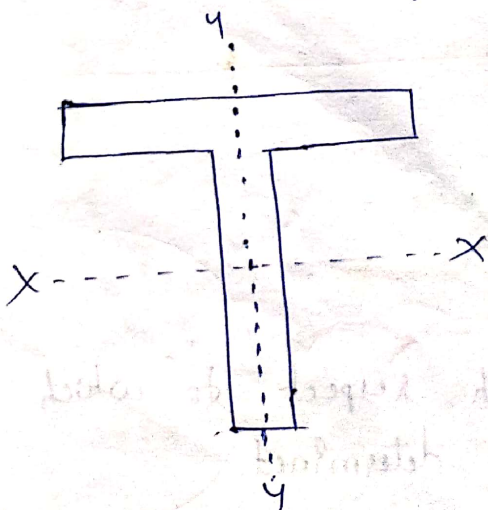
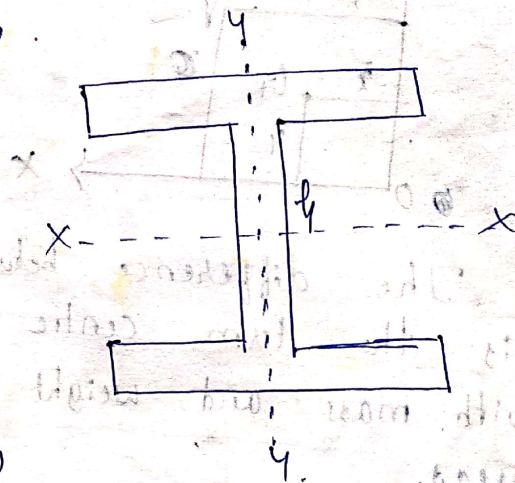
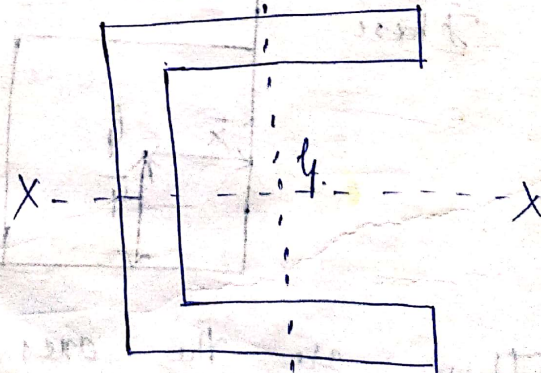


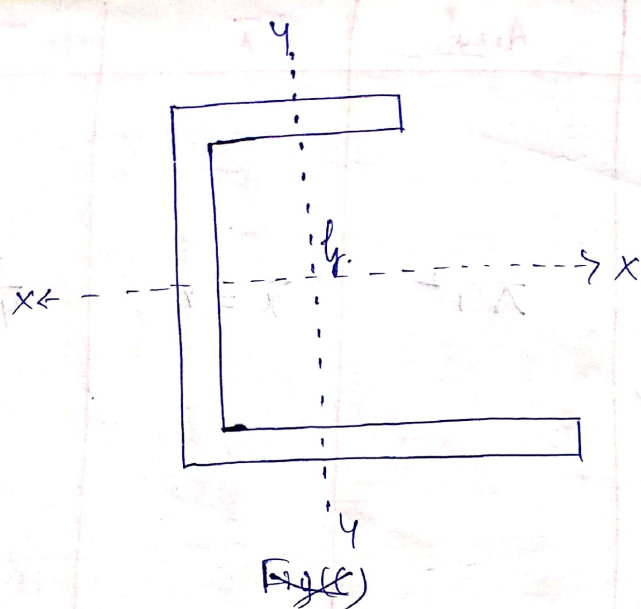
Fig (b)

Symmetry about the $y-y$ axis



Symmetry about the $x-x$ axis

Q.



Neither the x-x axis nor the y-y axis is the axis of symmetry

Centroids of some Important figures

Shape.	Area	$\bar{x}$	$\bar{y}$
Rectangle.	bd	$\frac{b}{2}$	$\frac{d}{2}$
Triangle.	$\frac{1}{2} b h$	$\frac{b}{2}$	$(\frac{1}{3}) h$ from base $(\frac{2}{3}) h$ from apex upper point
Right-angled triangle.	$\frac{1}{2} b h$	$(\frac{1}{3}) b$	$(\frac{1}{3}) h$

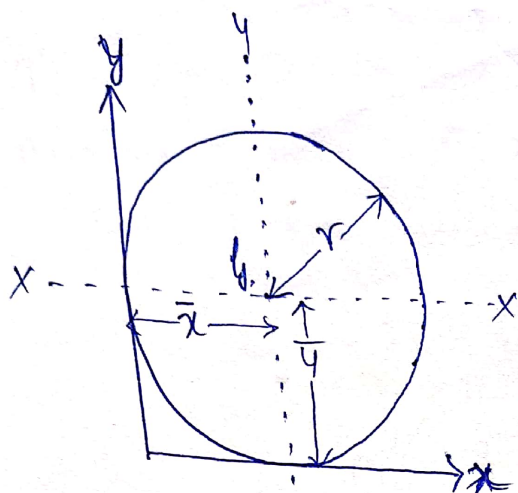
Shape

Area

$\bar{x}$

$\bar{y}$

Circle

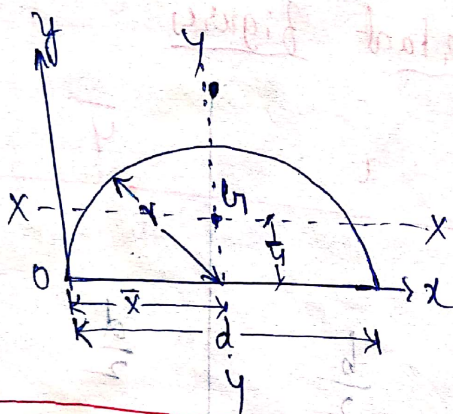


$$\pi r^2$$

$$\bar{x} = r$$

$$\bar{y} = r$$

Semicircle

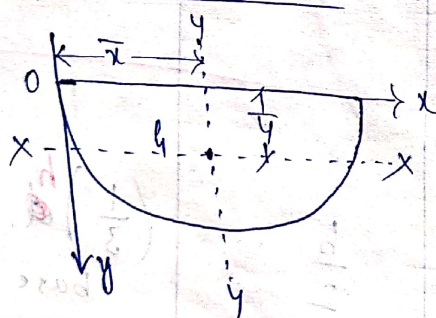


$$\frac{\pi r^2}{2}$$

$$\frac{d}{2}$$

$$\frac{4r}{3\pi}$$

Semicircle

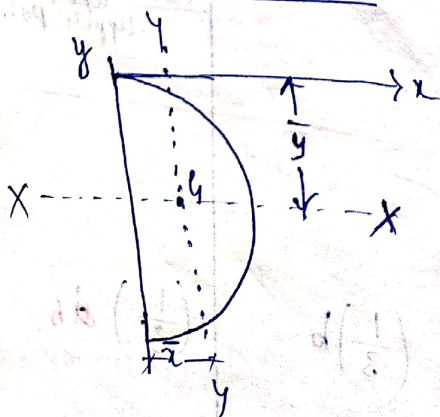


$$\frac{\pi r^2}{2}$$

$$\frac{d}{2}$$

$$-\frac{4r}{3\pi}$$

Semicircle



$$\frac{\pi r^2}{2}$$

$$\frac{4r}{3\pi}$$

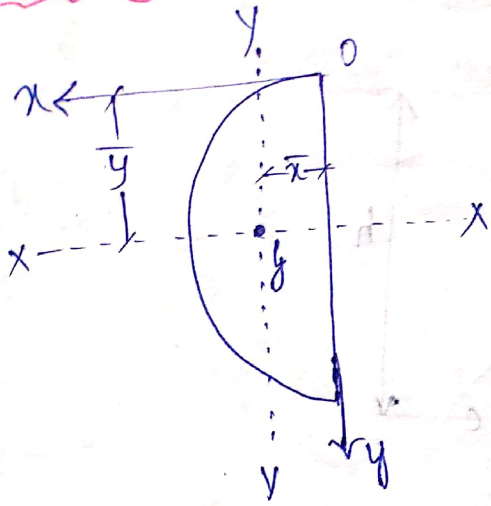
$$-\frac{d}{2}$$

Shape

Area

$\bar{x}$

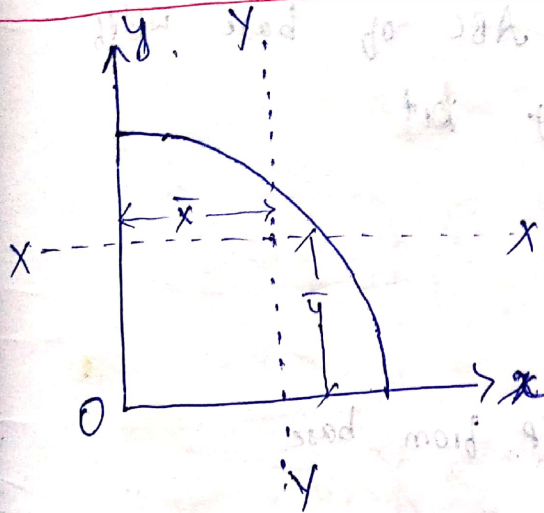
$\bar{y}$



$$\frac{\pi r^2}{2}$$

$$-\frac{4r}{3\pi}$$

$$-\frac{d}{2}$$



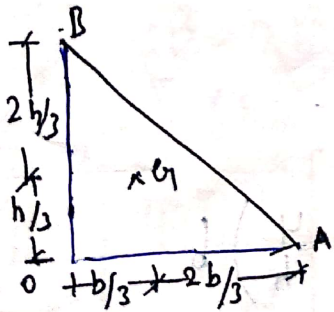
$$\frac{\pi r^2}{4}$$

$$\frac{4r}{3\pi}$$

$$\frac{4r}{3\pi}$$

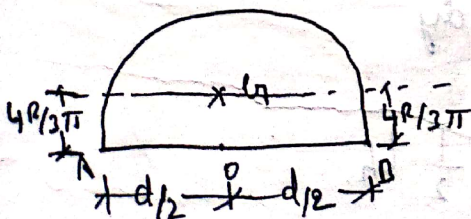
NOTE:-

(i)



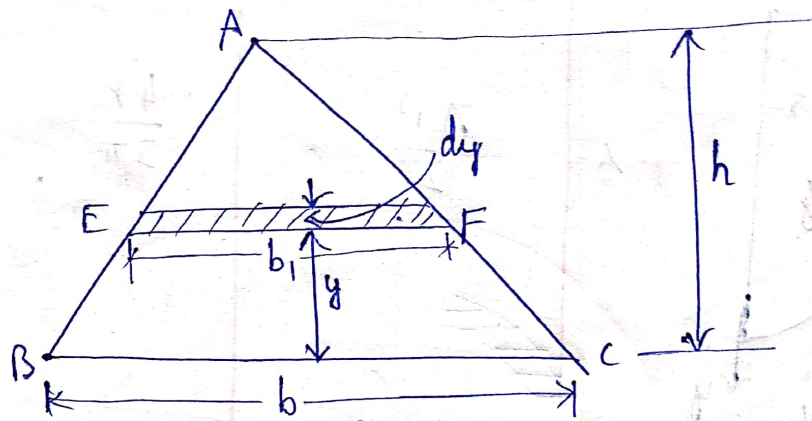
For the right angled triangle centroid always lies at a distance of $b/3$ or $h/3$ from the point where two $\perp$ lines intersect (Point O), and at a distance of $2h/3$ or $2b/3$ from other two points (Point A & B).

(ii)



For the semicircle centroid always lies at a distance of $\frac{4r}{3\pi}$ from the diametric axis, & at a distance of $d/2$ from either points (A & B).

Centroid of triangle :-



Consider the triangle ABC of base width 'b' and height 'h' as shown in fig. Let

b_1 = Width of elemental strip

dy = thickness of elemental strip

y = distance of elemental strip from base

Since $\triangle AEF$ & $\triangle ABC$ are the similar triangles,

$$\frac{b_1}{b} = \frac{h-y}{h}$$

$$b_1 = \left(\frac{h-y}{h} \right) b = \left(1 - \frac{y}{h} \right) b$$

Area of the element = $dA = b_1 dy$

$$dA = \left(1 - \frac{y}{h} \right) b dy$$

Area of the triangle $A = \frac{1}{2} bh$

Moment of area of strip about BC = Area of strip $\times y$

$$= \left(1 - \frac{y}{h} \right) b dy \times y$$

$$\bar{y} = \frac{\text{Moment of area}}{\text{Total area}} = \frac{\int y dA}{A}$$

Moment of elemental strip about base = $dA \cdot y$

∴ Moment of whole area = $\int_0^h y dA = \int_0^h y \left(1 - \frac{y}{h}\right) b dy$

$$= \int_0^h \left(y - \frac{y^2}{h}\right) b dy$$

$$= b \left[\frac{y^2}{2} - \frac{y^3}{3h} \right]_0^h$$

$$= b \left[\left(\frac{h^2}{2} - \frac{h^3}{3h} \right) - 0 \right]$$

$$= b \left[\frac{h^2}{2} - \frac{h^2}{3} \right]$$

$$= b \left[\frac{3h^2 - 2h^2}{6} \right] = b \left(\frac{h^2}{6} \right)$$

$$= b \left[\frac{h^2}{6} \right]$$

$$\int y dA = \frac{bh^2}{6}$$

$$\bar{y} = \frac{\int y dA}{A}$$

$$= \frac{\frac{bh^2}{6}}{\frac{bh}{2}}$$

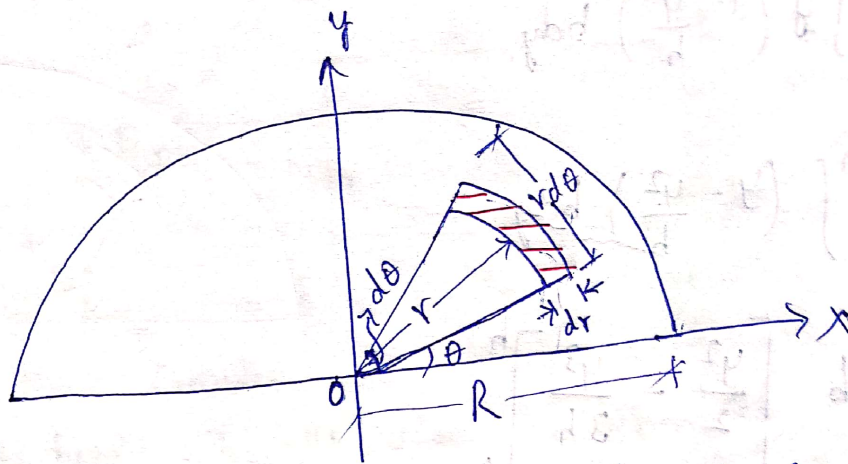
$$= \frac{bh^2}{6} \times \frac{2}{bh}$$

$$\boxed{\bar{y} = \frac{h}{3}}$$

Thus distance of centroid is at a distance of $\frac{h}{3}$ from the base @ $\frac{2h}{3}$ from the apex.

Centroid of semicircle

Consider the semicircle of radius



Consider the semicircle of radius 'R' as shown in fig. Due to symmetry centroid must lie on y-axis. Let its distance from diametral axis be $\bar{y}$. To find $\bar{y}$ consider an elemental strip at a distance 'r' from the centre 'O' of the semicircle, radial width being dr and bound by radii θ & $\theta + d\theta$.

The elemental area may be treated as a rectangle of sides $r d\theta$ & dr .

$$\therefore \text{Area of element} = r d\theta \cdot dr$$

Moment of area of element about x-axis is

$$= r d\theta \cdot dr \times r \sin \theta$$

$$= r^2 \sin \theta dr d\theta.$$

$\therefore$ Total moment of area about diametral axis,

$$= \int_0^\pi \int_0^R r^2 \sin \theta dr d\theta.$$

$$= \int_0^\pi \left[\frac{r^3}{3} \right]_0^R \sin \theta d\theta$$

$$\left[\frac{r^3}{3} \right]_0^R = \left[\frac{r^3}{3} \right]_0^R$$

$$= \int_0^{\pi} \frac{R^3}{3} \sin \theta d\theta$$

$$= \frac{R^3}{3} \int_0^{\pi} \sin \theta d\theta$$

$$= \frac{R^3}{3} [-\cos \theta]_0^{\pi} = -\frac{R^3}{3} [\cos \theta]_0^{\pi}$$

$$= -\frac{R^3}{3} [(\cos \pi) - (\cos 0)]$$

$$= -\frac{R^3}{3} [-1 - 1]$$

$$= \frac{2R^3}{3}$$

Area of semicircle $A = \frac{1}{2} \pi R^2$

$$\bar{y} = \frac{\text{Moment of area}}{\text{Total area.}}$$

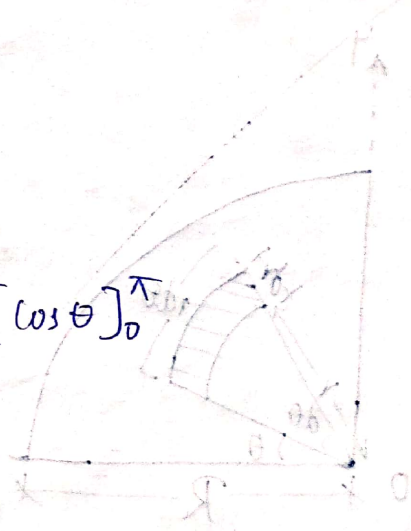
$$\bar{y} = \frac{\frac{2R^3}{3}}{\frac{\pi R^2}{2}}$$

$$\bar{y} = \frac{2R^3}{3} \times \frac{2}{\pi R^2}$$

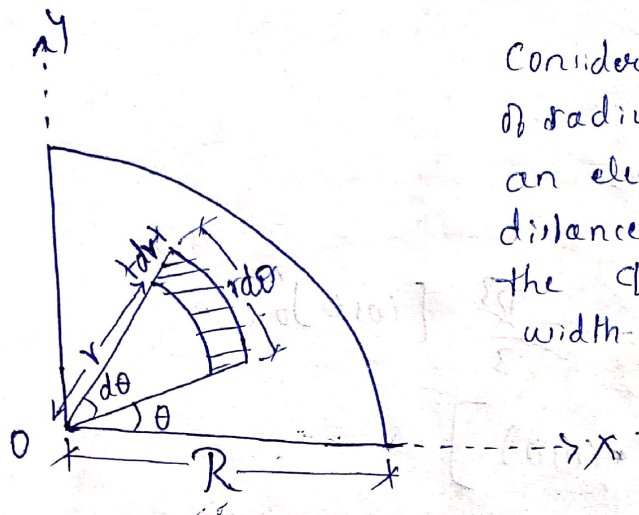
$$\boxed{\bar{y} = \frac{4R}{3\pi}}$$

Thus, the centroid of the ^{semi}circle is at a distance

$\frac{4R}{3\pi}$ from the diametrical axis.



Centroid of Quadrant of a circle



Consider the quadrant of a circle of radius 'R' as shown in fig. Consider an elemental strip at a radial distance 'r' from the centre 'O' of the quadrant of the circle, radial width being 'dr' & bound by radii θ & $\theta + d\theta$.

Moment about diametral x-axis is given by $\int_0^{\pi/2} \int_0^R r^2 \sin \theta dr d\theta$

[Note: Change from that of a semicircle is only in the limit of integration for θ].

$$\int_0^{\pi/2} \int_0^R r^2 \sin \theta dr d\theta$$

$$= \int_0^{\pi/2} \left[\frac{r^3}{3} \right]_0^R \sin \theta d\theta$$

$$= \frac{R^3}{3} \int_0^{\pi/2} \sin \theta d\theta$$

$$= \frac{R^3}{3} [-\cos \theta]_0^{\pi/2}$$

$$= -\frac{R^3}{3} \left[\cos \frac{\pi}{2} - \cos 0 \right]$$

$$= -\frac{R^3}{3} [0 - 1]$$

$$= \frac{R^3}{3}$$

Area of quadrant of a circle = $\frac{1}{4} \pi R^2$

$$\bar{y} = \frac{\text{Moment of area}}{\text{Total area.}}$$

$$\bar{y} = \frac{\frac{R^3}{3}}{\frac{\pi R^2}{4}}$$

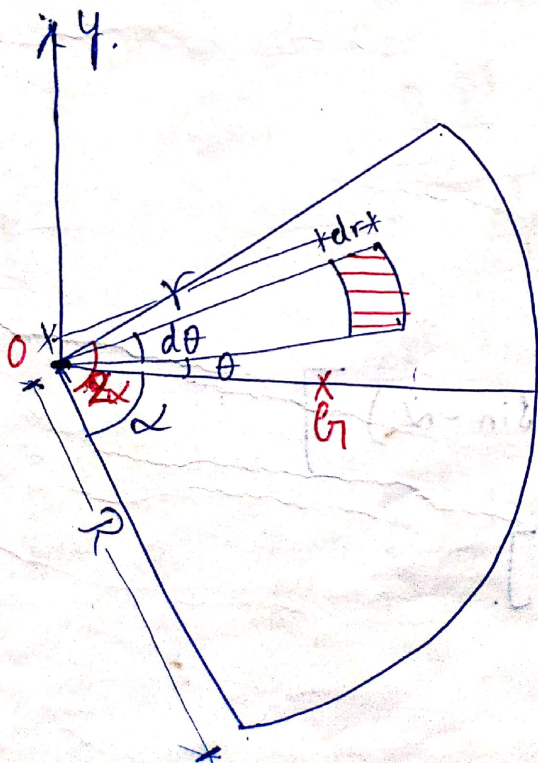
$$\bar{y} = \frac{R^3}{3} \times \frac{4}{\pi R^2}$$

$$\boxed{\bar{y} = \frac{4R}{3\pi}}$$

Similarly by taking moment of area about y-axis

$$\boxed{\bar{x} = \frac{4R}{3\pi}}$$

Centroid of Sector of a Circle



Consider the sector of a circle of angle 2α as shown in fig. Due to symmetry centroid lies on x-axis. To find its distance from the centre 'O', consider the elemental area shown.

$$\text{Area of the elemental strip} = r d\theta \cdot dr$$

Moment of area of the elemental strip about y-axis.

$$= r d\theta \cdot dr \cdot r \cos\theta$$

$$= r^2 \cos\theta \cdot dr d\theta$$

$\therefore$ Total moment of area about y-axis

$$= \int_{-\alpha}^{\alpha} \int_0^R r^2 \cos\theta dr d\theta$$

$$= \int_{-\alpha}^{\alpha} \left[\frac{r^3}{3} \right]_0^R \cos\theta d\theta$$

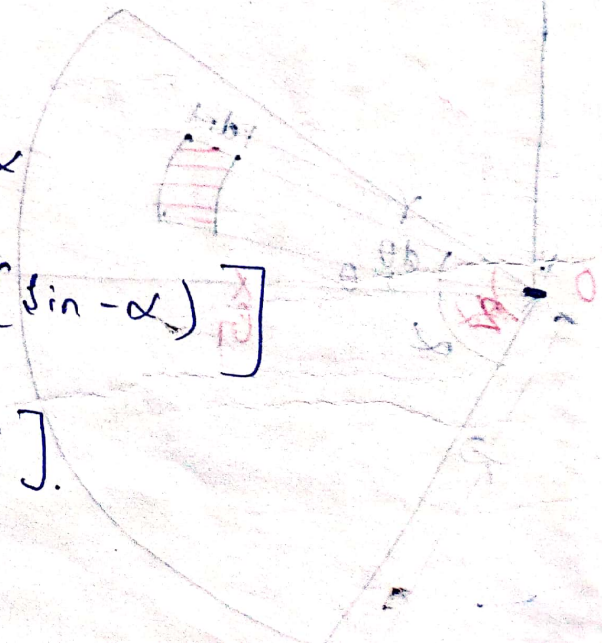
$$= \left[\frac{R^3}{3} \right]_{-\alpha}^{\alpha} \cos\theta d\theta$$

$$= \frac{R^3}{3} \left[+\sin\theta \right]_{-\alpha}^{\alpha}$$

$$= +\frac{R^3}{3} \left[\sin\alpha - (\sin - \alpha) \right]$$

$$= +\frac{R^3}{3} \left[2 \sin\alpha \right]$$

$$= \frac{2R^3}{3} \sin\alpha$$



Total area of the sector

$$= \int_{-\alpha}^{\alpha} \int_0^R r d\theta \cdot dr$$

$$= \int_{-\alpha}^{\alpha} \left[\frac{r^2}{2} \right]_0^R d\theta$$

$$= \frac{R^2}{2} \int_{-\alpha}^{\alpha} d\theta$$

$$= \frac{R^2}{2} [\theta]_{-\alpha}^{\alpha}$$

$$= \frac{R^2}{2} [\alpha - (-\alpha)]$$

$$= \frac{2R^2\alpha}{2}$$

$$= \underline{\underline{R^2\alpha}}$$

The distance of centroid from centre 'O'

$$= \frac{\text{Moment of area about y-axis}}{\text{Area of the fig.}}$$

$$= \frac{\frac{2R^3}{3} \sin\alpha}{\frac{R^2\alpha}{1}}$$

$$= \frac{2R^3}{3} \sin\alpha \times \frac{1}{R^2\alpha}$$

Dist. of Centroid from 'O' = $\frac{2R \sin\alpha}{3\alpha}$

Centre of gravity of a plane fig.

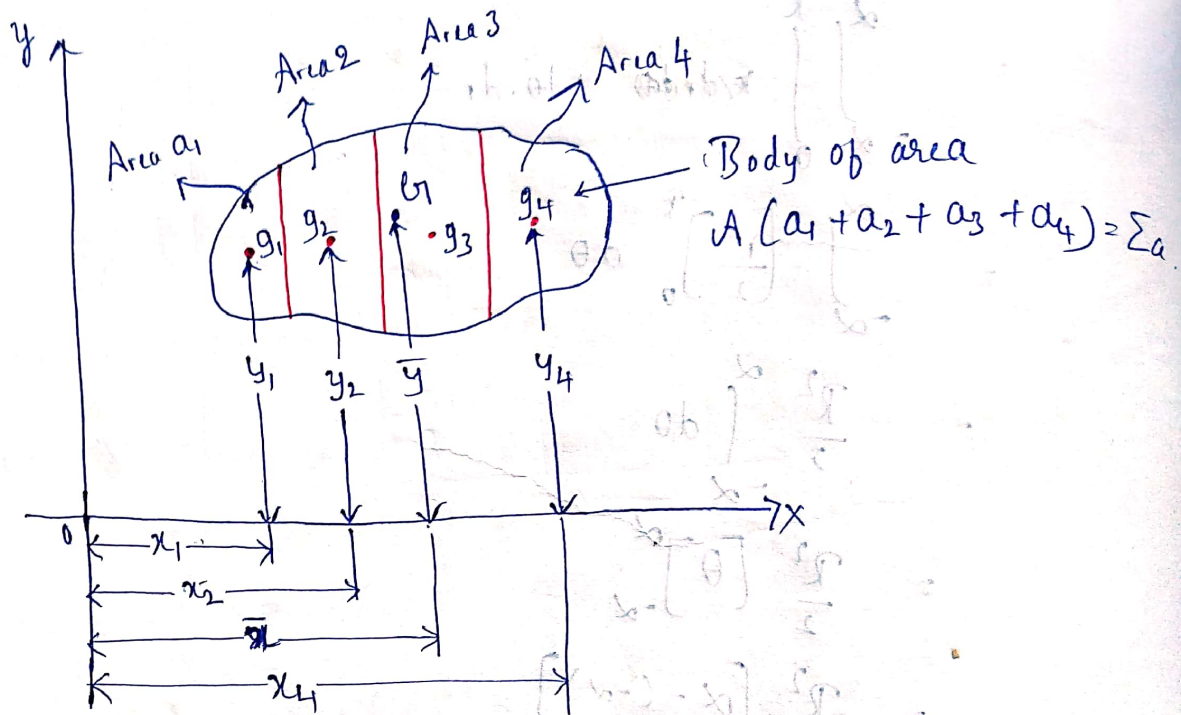


Fig. shows a plane figure of total area A and unit thickness. Let 'y' be the C.G. of whole area with coordinates $(\bar{x}, \bar{y})$. Divide this area 'A' into a number of small areas a_1, a_2, a_3 & a_4 with centroids g_1, g_2, g_3 & g_4 respectively.

$$A = \Sigma a = a_1 + a_2 + a_3 + a_4 \longrightarrow (1)$$

Let $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ & (x_4, y_4) be their co-ordinates respectively.

Moment of the total area 'A' about 'y' axis.

$$A \bar{x} = \Sigma a \bar{x} \longrightarrow (2)$$

From varignon's theorem.

Sum of moments of all areas about

$$A \bar{x} = a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4$$

$$\Sigma a \cdot \bar{x} = \Sigma a x \longrightarrow (3)$$

$$\Sigma a \cdot \bar{x} = \Sigma a x$$

$$\boxed{\bar{x} = \frac{\Sigma a x}{\Sigma a}}$$

∴ y axis is equal to the moment of whole area about the same axis.

Steps in solving the problems:-

- * Identify whether the given fig. is symmetric about x -axis & y -axis.
- * If the given fig. is symmetric about y -axis then $\bar{x}$ is known. & if the fig. is symmetric about x -axis then $\bar{y}$ is known.
- * Draw the reference axis.
- * Divide the given fig. into a number of plane areas (Δ^k , $\square^k$, semicircle, quadrant of a circle) respectively.
- * Enter the values in the tabular column given below.

a.) If the given fig. is symmetric about only y -axis & not symmetric about x -axis then

Component	Area (a)	Centroidal dist. from x -axis (y)	Centroidal ay
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$$\bar{y} = \frac{\sum ay}{\sum a}$$

b.) If the fig. is symmetric about only x -axis & not symmetric about y -axis then

Component	Area (a)	Centroidal dist. from y -axis (x)	ax
-----------	----------	---	------

$$\bar{x} = \frac{\sum ax}{\sum a}$$

c.) If the fig. is not symmetric about both x & y axis.

Component	Area (a)	Centroidal dist. from y -axis (x)	Centroidal dist. from x -axis (y)	ax	ay
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$$\bar{x} = \frac{\sum ax}{\sum a}$$

$$\bar{y} = \frac{\sum ay}{\sum a}$$

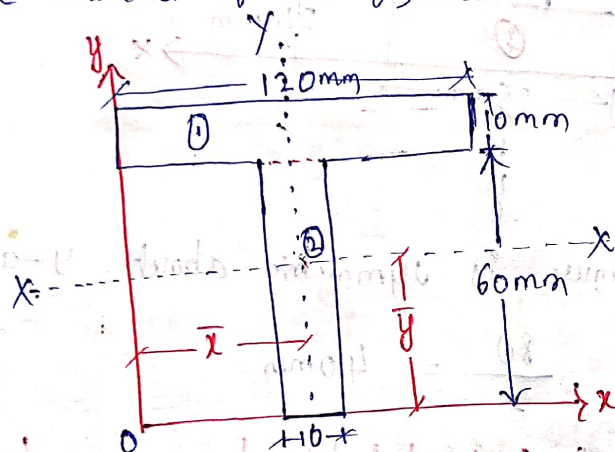
Similarly,

$$\sum a\bar{y} = \sum ay$$

$$\bar{y} = \frac{\sum ay}{\sum a}$$

Problems:-

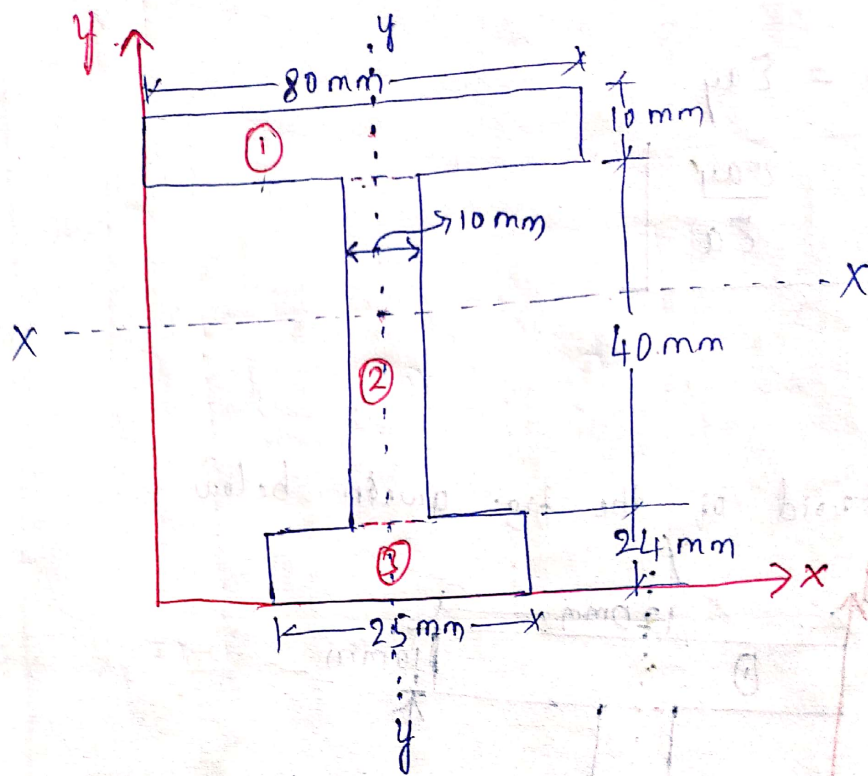
1. Find the centroid of the fig. given below



Component	Area, a (mm^2)	Centroidal distance from y-axis ($\bar{x}$)	Centroidal distance from x-axis ($\bar{y}$)	$a\bar{x}$	$a\bar{y}$
Rectangle ①	$A = b \times d$ $120 \times 10 = 1200$	$\bar{x} = b/2$ $\frac{120}{2} = 60$	$y = d/2$ $60 + \frac{10}{2} = 65$	72000	78000
Rectangle ②	$10 \times 60 = 600$	$\frac{120}{2} = 60$	$\frac{60}{2} = 30$	36000	18000
Sum	$\sum a = 1800$			108000	96000
	$\bar{x} = \frac{\sum a\bar{x}}{\sum a} = \frac{108000}{1800} = 60 \text{ mm}$				
	$\bar{y} = \frac{\sum a\bar{y}}{\sum a} = \frac{96000}{1800} = 53.33 \text{ mm}$				

NOTE:- Given figure is symmetric about y-axis, therefore we can write directly $\bar{x} = \frac{120}{2} = 60 \text{ mm}$

2-5 Find the centroid for the fig. given below.



Soln:- The given figure is symmetric about y-axis,

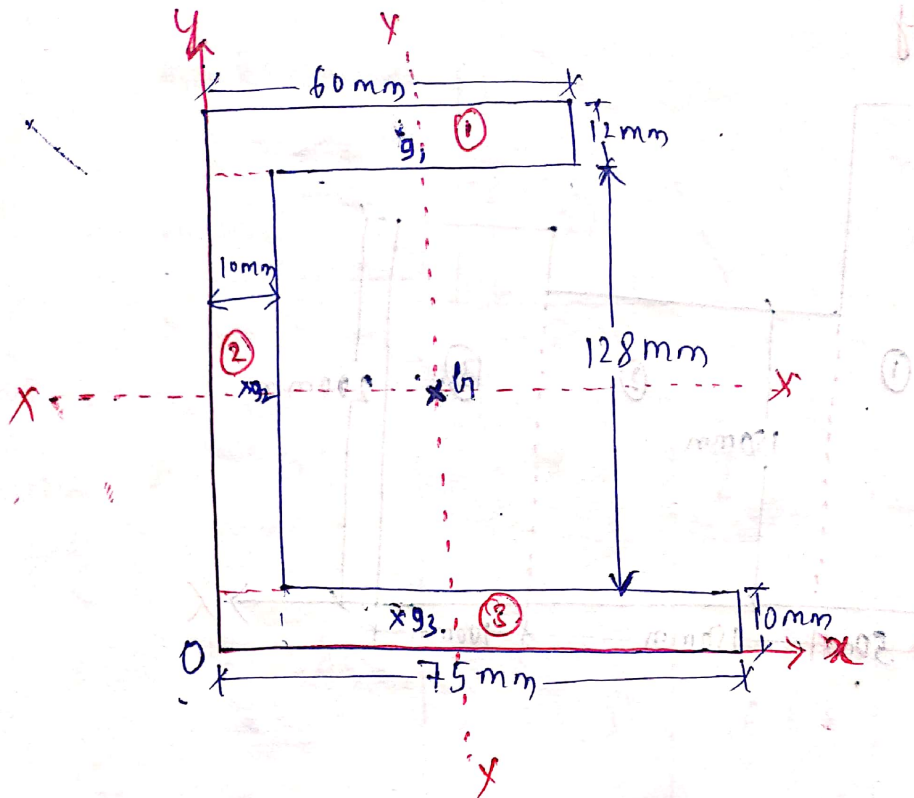
Hence $\bar{x} = \frac{80}{2} = 40 \text{ mm}$.

Component	Area, a (mm^2)	Centroidal distance from x-axis (y)	ay
Rectangle ①	$10 \times 80 = 800$	$24 + 40 + \frac{10}{2} = 69$	55,200
Rectangle ②	$10 \times 40 = 400$	$24 + \frac{40}{2} = 44$	17,600
Rectangle ③	$25 \times 24 = 600$	$\frac{24}{2} = 12$	7,200
$\Sigma a = 1800$			$\Sigma ay = 80,000$

$$\bar{y} = \frac{80,000}{1800} = 44.44 \text{ mm}$$

$$\bar{x} = 40 \text{ mm}$$

3. } Determine the centroid for the fig. given below.

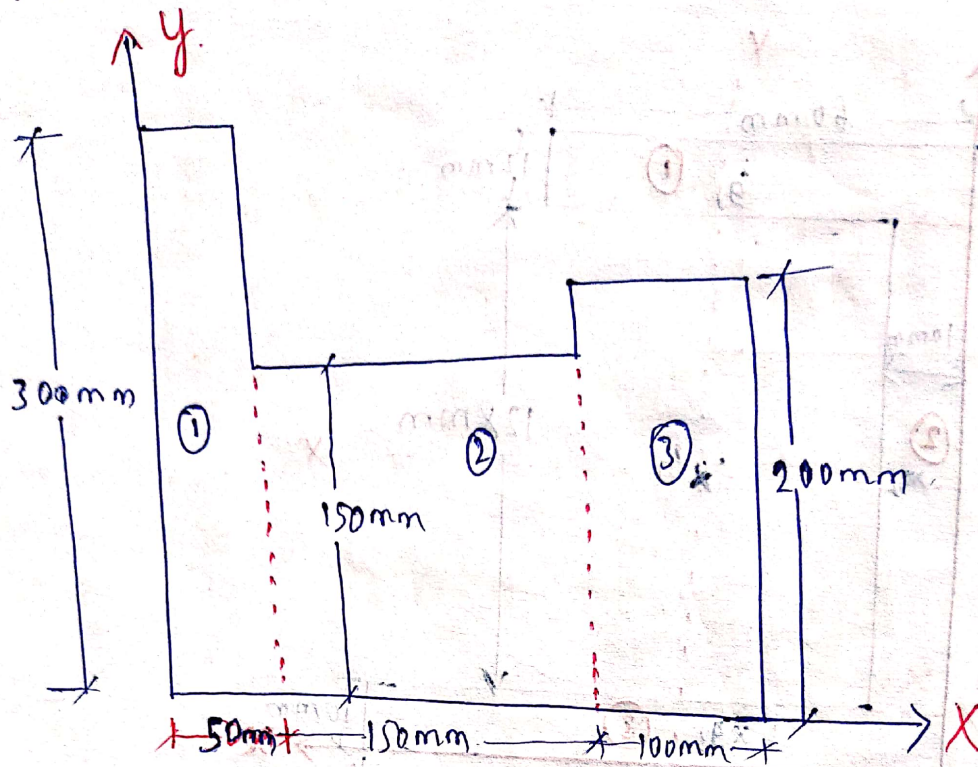


Component	Area, a mm^2	x	y	ax	ay
$\square^1$ ①	$60 \times 12 = 720$	$\frac{60}{2} = 30$	$10 + 128 + \frac{12}{2} = 144$	21600	103680
$\square^1$ ②	$10 \times 128 = 1280$	$\frac{10}{2} = 5$	$10 + \frac{128}{2} = 74$	6400	94720
$\square^1$ ③	$75 \times 10 = 750$	$\frac{75}{2} = 37.5$	$\frac{10}{2} = 5$	28125	3750
Sum	$\Sigma a = 2750$			$\Sigma ax = 56125$	$\Sigma ay = 202150$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{56125}{2750} = 20.4 \text{ mm}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{202150}{2750} = 73.5 \text{ mm}$$

4.} Determine the centroid of for the fig. given below.



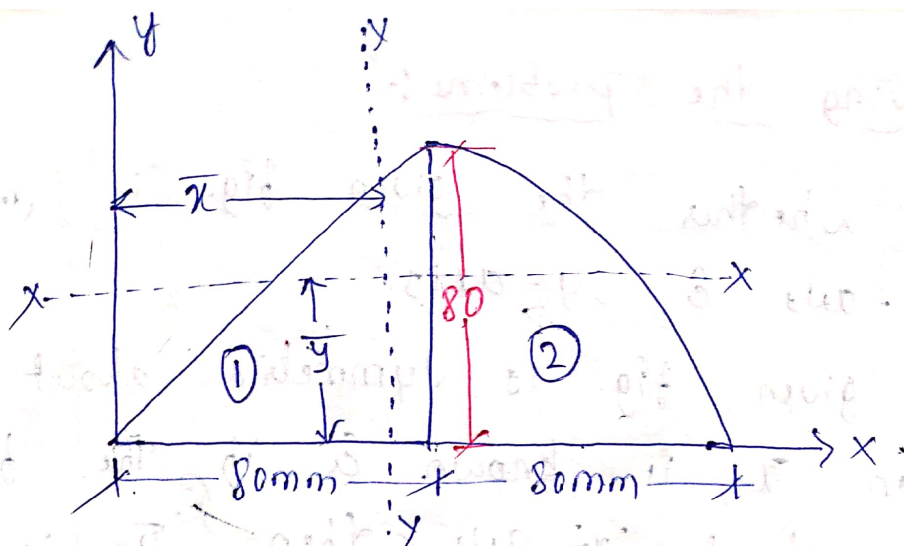
Soln:

Component Rect	Area (a) mm ²	x	y	ax	ay
Rect ①	$50 \times 300 = 15000$	$\frac{50}{2} = 25$	$\frac{300}{2} = 150$	375000	2250000
Rect ②	$150 \times 150 = 22500$	$50 + \frac{150}{2} = 125$	$\frac{150}{2} = 75$	2812500	1687500
Rect ③	$100 \times 200 = 20000$	$50 + 150 + \frac{100}{2} = 250$	$\frac{200}{2} = 100$	2000000	2000000
Sum	$\Sigma a = 57500$			$\Sigma ax = 8187500$	$\Sigma ay = 5937500$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{8187500}{57500} = 142.4 \text{ mm}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{5937500}{57500} = 103.26 \text{ mm}$$

5.8



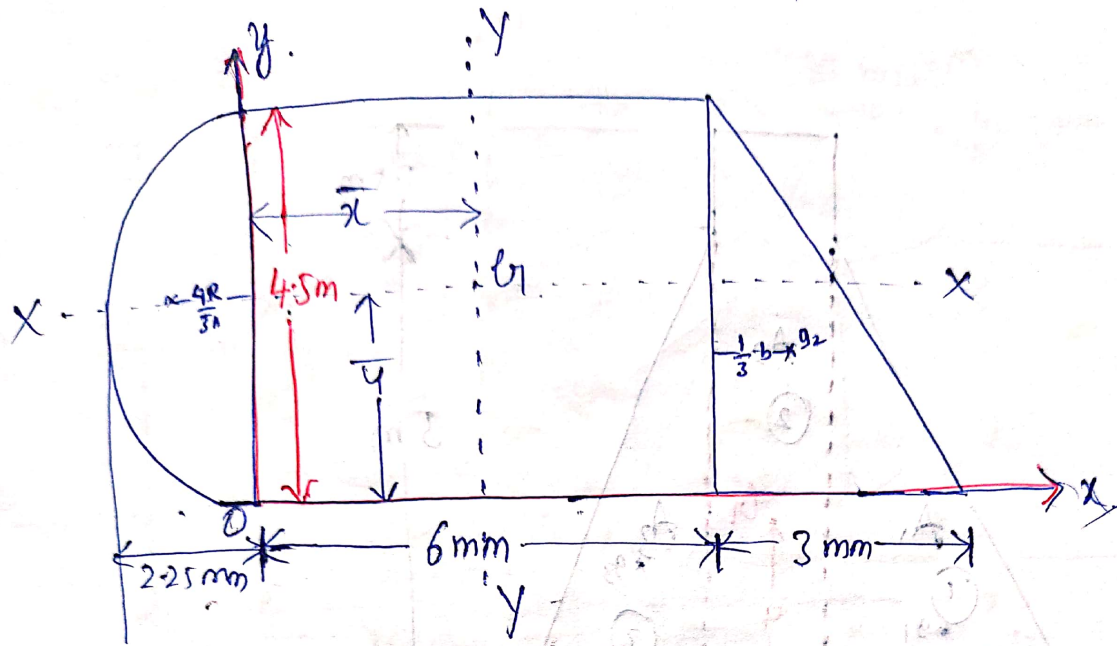
Component	Area, (a) mm ²	$\bar{x}$	$\bar{y}$	$a\bar{x}$	$a\bar{y}$
Δ ^{le} ①	$\frac{1}{2} \times 80 \times 80$ = 3200	$\frac{2}{3} \times 80$ = 53.33	$\frac{1}{3} \times 80$ = 26.67	170656.6	85,333.33
Quarter circle ②	$\frac{\pi r^2}{4} = \frac{\pi \times 80^2}{4}$ = 5026.55	$80 + \frac{4 \times 80}{3 \times \pi}$ = 113.953	$\frac{4 \times 80}{3 \times \pi}$ = 33.953	572790.45 170666.45	170666.72
	$\Sigma a = 8226.55$		$\Sigma \bar{x}$	743447.05 = 341329.05	$\Sigma a\bar{y} =$ 256000.78

$$\bar{x} = \frac{\Sigma a\bar{x}}{\Sigma a} = \frac{743447.05}{8226.55}$$

$$\bar{x} = 90.373 \text{ mm}$$

$$\bar{y} = \frac{\Sigma a\bar{y}}{\Sigma a} = \frac{256000.78}{8226.55} = 31.119 \text{ mm}$$

6) Calculate the centroid of fig. given below with to 'O'.

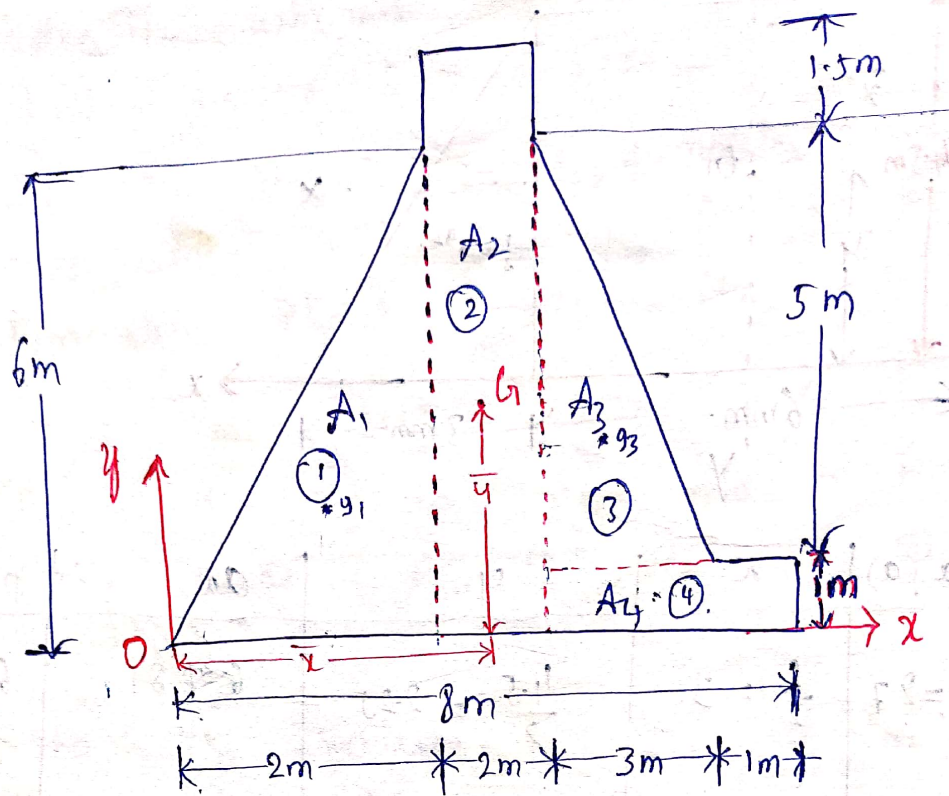


Component	Area (a)	x	y	ax	ay
Rectangle	$6 \times 4.5 = 27$	$\frac{6}{2} = 3$	$\frac{4.5}{2} = 2.25$	60.75	60.75
Δ	$\frac{1}{2} \times 3 \times 4.5 = 6.75$	$6 + \frac{1}{3} \times 3 = 7$	$\frac{1}{3} \times 4.5 = 1.5$	47.25	10.125
Semicircle	$\frac{\pi \times (2.25)^2}{2} = 7.95$	$-\frac{4 \times 2.25}{3\pi} = -0.955$	$\frac{4.5}{2} = 2.25$	-7.59	17.88
				$\Sigma ax = 120.66$	$\Sigma ay = 88.755$

$$\Sigma a = 41.7$$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{120.66}{41.7} = 2.893 \text{ mm}$$

$$\bar{y} = \frac{88.755}{41.7} = 2.129 \text{ mm}$$

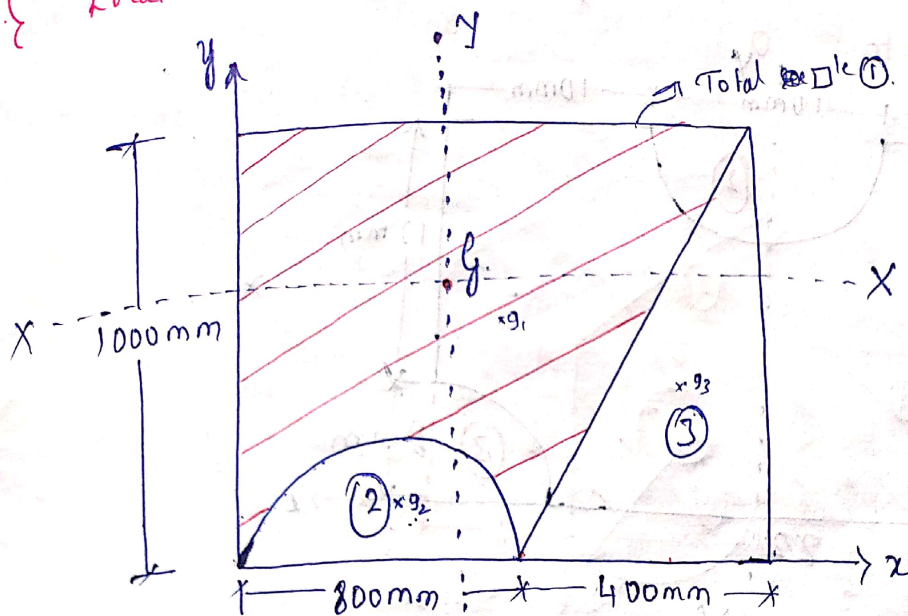


Component	Area (a)	x	y	ax	ay
Δ^{le} ①	$\frac{1}{2} \times 2 \times 6 = 6$	$\frac{2}{3} \times 6 = \frac{2}{3} \times 2 = 1.33\text{m}$	$\frac{1}{3} \times 6 = 2\text{m}$	7.98	12
$\square^{\text{le}}$ ②	$2 \times (1.5 + 5 + 1) = 15$	$2 + \frac{2}{2} = 3\text{m}$	$\frac{1.5 + 5 + 1}{2} = 3.75$	45	56.25
Δ^{le} ③	$\frac{1}{2} \times 3 \times 5 = 7.5$	$2 + 2 + \frac{1}{3} \times 3 = 5\text{m}$	$1 + \frac{1}{3} \times 5 = 2.67$	18.75	20.025
$\square^{\text{le}}$ ④	$4 \times 1 = 4$	$2 + 2 + \frac{4}{2} = 6$	$\frac{1}{2} = 0.5$	24	2
$\Sigma a = 32.5$				$\Sigma ax = 114.5$	$\Sigma ay = 90.275$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{114.5}{32.5} = 3.52\text{m}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{90.275}{32.5} = 2.77\text{m}$$

7.1 Locate the centroid of the shaded area shown in fig.



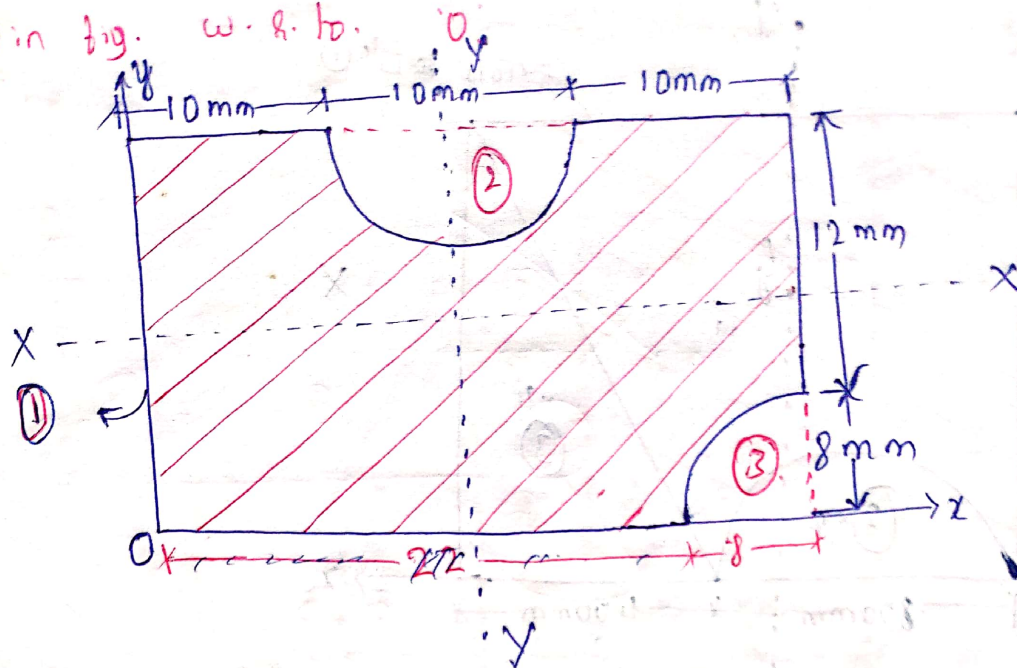
Soln:

Component	Area (a) mm ²	x	y	ax	ay
Rect ①	$1000 \times (800 + 400)$ $= 1200000$	$\frac{1200}{2} = 600$	$\frac{1000}{2} = 500$	720000000	600000000
Semicircle ②	$\frac{\pi R^2}{2} = \frac{\pi \times 400^2}{2} = 400\pi$ $= 251327.41$	$R = \frac{800}{2} = 400$	$\frac{4R}{3\pi} = \frac{4 \times 400}{3\pi} = 169.76$	-100530964	-42665341.12
Tri ③	$\frac{1}{2} \times 400 \times 1000$ $= 200000$	$800 + \frac{2b}{3} = 800 + 2 \times \frac{400}{3} = 1066.67$	$\frac{h}{3} = \frac{1000}{3} = 333.33$	-213334000	-66666666.6
$\Sigma a = 748672.6$		$\Sigma ax = 40613503.6$		$\Sigma ay = 490667992.2$	

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{40613503.6}{748672.6} = 542.473 \text{ mm}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{490667992.2}{748672.6} = 655.38 \text{ mm}$$

8) Determine the centre of gravity of the lamina shown in fig. w.r.to.

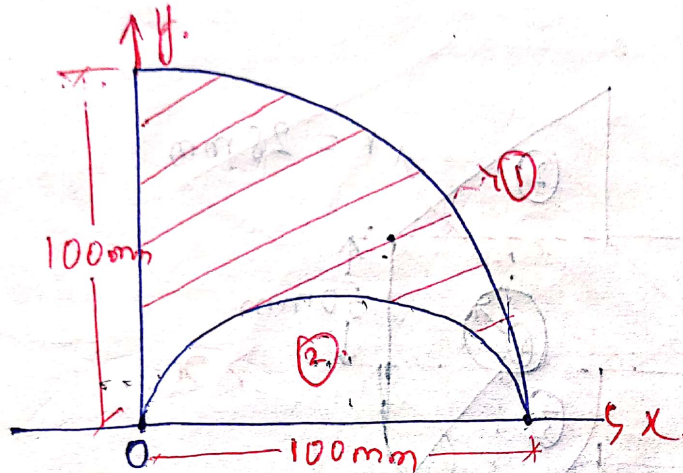


Component	Area(a) mm ²	x	y	ax	ay
Total or ①	$(10+10+10) \times (12+8)$ $= 600$	$\frac{10+10+10}{2}$ $= 15$	$\frac{12+8}{2} = 10$	9000	6000
Semicircle ②	$\frac{\pi R^2}{2} = \frac{\pi \times 5^2}{2}$ $= -39.27$	$10 + R = 10 + 5$ $= 15$	$(12+8) - \frac{4R}{3\pi}$ $= 20 - \frac{4 \times 5}{3\pi}$ $= 17.87$	-589.05	-701.75
Quadrant of the circle ③	$\frac{\pi R^2}{4} = \frac{\pi \times 8^2}{4}$ $= -50.26$	$30 - \frac{4R}{3\pi}$ $30 - \frac{4 \times 8}{3\pi}$ $= 26.6$	$\frac{4R}{3\pi} = \frac{4 \times 8}{3\pi}$ $= 3.395$	-1337.15	+170.63
Σa	510.47			$\Sigma ax = 7073.85$	$\Sigma ay = 5127.62$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{7073.85}{510.47} = 13.85 \text{ mm}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{5127.62}{510.47} = 10.04 \text{ mm}$$

Q. Find the centroid of the shaded area shown in fig. obtained by cutting a semicircle of diameter 100mm from the quadrant of a circle of radius 100mm.



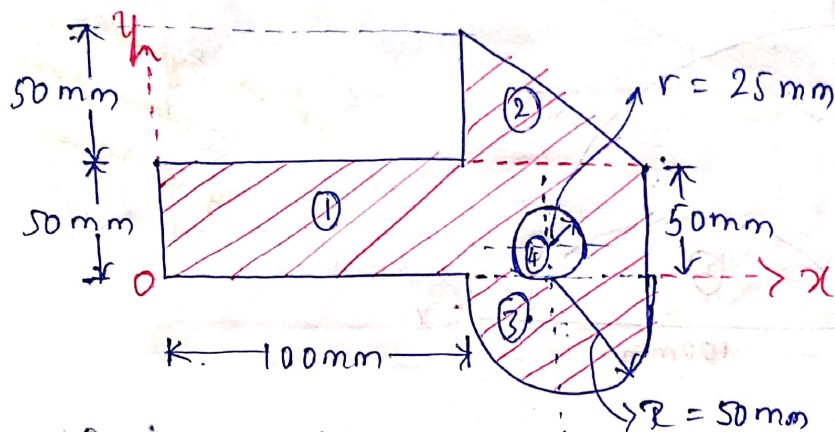
Component	Area (mm ²)	x	y	a _x	a _y
Total quadrant of the circle	$\frac{\pi R^2}{4} = \frac{\pi \times 100^2}{4}$ = 7854	$\frac{4R}{3\pi} = \frac{4 \times 100}{3\pi}$ = 42.44	$\frac{4R}{3\pi} = \frac{4 \times 100}{3\pi}$ = 42.44	333323.76	333323.76
Semicircle	$\frac{\pi R^2}{2} = \frac{\pi \times 50^2}{2}$ = -3926.99	R = 50	$\frac{4R}{3\pi} = \frac{4 \times 50}{3\pi}$ = 21.22	-196349.5	-83330.72
	$\Sigma a = 3927.01$			$\Sigma a_x = 136974.26$	$\Sigma a_y = 249993.04$

$$\bar{x} = \frac{\Sigma a_x}{\Sigma a} = \frac{136974.26}{3927.01}$$

$$= \underline{\underline{34.88 \text{ mm}}}$$

$$\bar{y} = \frac{\Sigma a_y}{\Sigma a} = \frac{249993.04}{3927.01} = \underline{\underline{63.66 \text{ mm}}}$$

10. Determine the location of the centroid of the shaded portion of the lamina, shown in fig. w.r.to. the origin 'O'

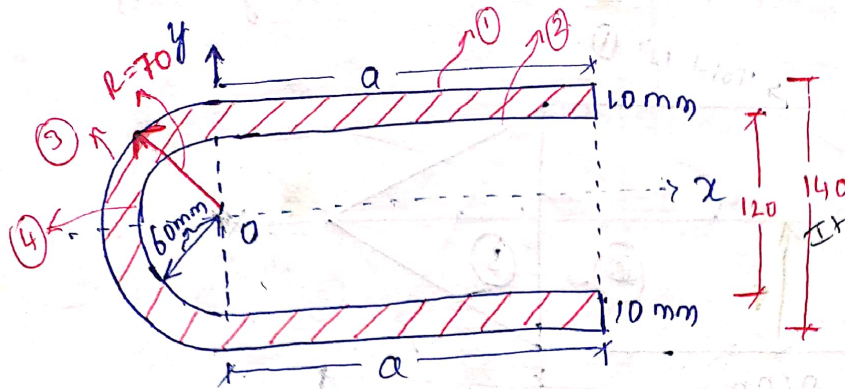


Component	Area (a) mm ²	$\bar{x}$ mm	$\bar{y}$ mm	$a\bar{x}$	$a\bar{y}$
Rect (1)	$(100+50+50) \times 50$ $= 10000$	$\frac{100+100}{2} = 100$	$\frac{50}{2} = 25$	1000000	250000
Δ (2)	$\frac{1}{2} \times 100 \times 50$ $= 2500$	$100 + \frac{b}{3}$ $= 100 + \frac{100}{3}$ $= 133.33$	$50 + \frac{h}{3}$ $= 50 + \frac{50}{3} = 66.67$	333325	166675
Semi-circle (3)	$\frac{\pi R^2}{2} = \frac{\pi \times 50^2}{2}$ $100 + R = 150$ $= 3927.5$	$100 + R = 150$	$-\frac{4R}{3\pi}$ $= -\frac{4 \times 50}{3\pi}$ $= -21.22$	589125	-83341.55
Circle (4)	$\pi r^2 = \pi \times 25^2$ $= 1963.5$	$100 + 50 = 150$	$r = 25$	294525	-49087.5
	$\Sigma a = 14463.5$			$\Sigma a\bar{x} = 1627925$	$\Sigma a\bar{y} = 284245.95$

$$\bar{x} = \frac{\Sigma a\bar{x}}{\Sigma a} = \frac{1627925}{14463.5} = 112.55 \text{ mm}$$

$$\bar{y} = \frac{\Sigma a\bar{y}}{\Sigma a} = \frac{284245.95}{14463.5} = 19.65 \text{ mm}$$

1st For the shaded area shown in fig, find 'a' so that centroid is at 'O'.



is symmetric about x-axis
y is known

Given data:-

Since centroid is lying at 'O' so $\bar{x} = 0$,
 1) For outer $\Delta^e \Rightarrow R = 60 + 10 = 70$. For inner $\Delta^e \Rightarrow r = 60$.
 $a = ?$, 2) For inner $\Delta^e \Rightarrow a = 60$. For outer $\Delta^e \Rightarrow b = a$, $d = 60 + 10 + 10 = 80$.
 $\Rightarrow b = a$, $d = 60$ mm.

Component	Area (a) mm ²	$\bar{x}$	$a\bar{x}$
Rect ① [Outer]	$a \times [10 + 60 + 60 + 10]$ $= 140a$	$\frac{a}{2}$	$70a^2$
Rect ② [Inner]	$-a \times [60 + 60]$ $= -120a$	$\frac{a}{2}$	$-60a^2$
Semicircle ③ [Outer]	$\frac{\pi R^2}{2} = \frac{\pi (60+10)^2}{2}$ 7696.9	$-\frac{4R}{3\pi} = -\frac{4 \times 70}{3\pi}$ $= -29.7$	-228597.93
Semicircle ④ [Inner]	$-\frac{\pi R^2}{2} = -\frac{\pi \times 60^2}{2}$ $= -5654.86$	$-\frac{4R}{3\pi} = -\frac{4 \times 60}{3\pi}$ $= -25.465$	14400.1
Σa	$140a - 120a + 7696.9 - 5654.86$		

$$\Sigma a\bar{x} = 70a^2 - 60a^2 - 228597.93 + 14400.1$$

$$\Sigma a\bar{x} = 10a^2 - 84596.93$$

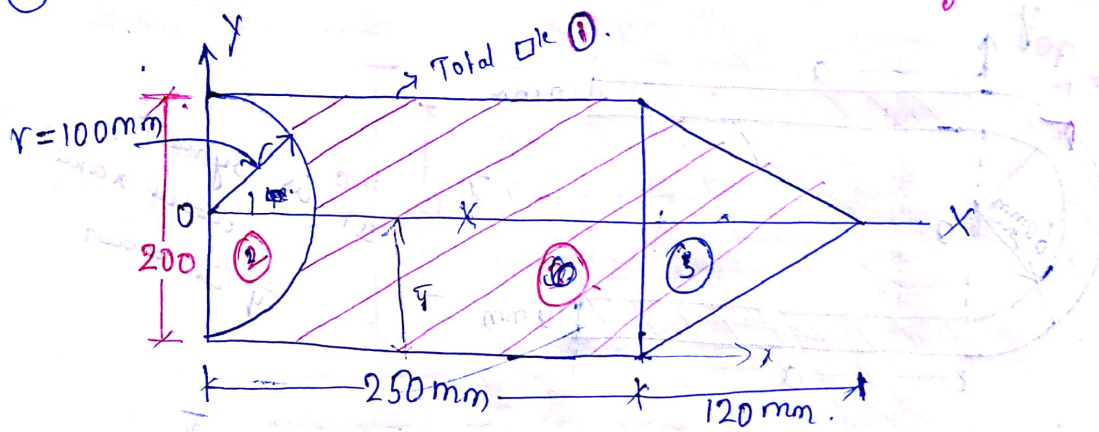
$$\bar{x} = \frac{\Sigma a\bar{x}}{\Sigma a} = \frac{10a^2 - 84596.93}{140a - 120a + 7696.9 - 5654.86} = 0$$

$$\Rightarrow 10a^2 - 84596.93 = 0$$

$$a^2 = 8459.693$$

$$a = 91.97 \text{ mm}$$

125 Find C.G. of the shaded area for the given fig. with respect to given x & y axes.



The given fig. is symmetric about x-axis.

$\therefore \bar{y}$ is known.

$$\bar{y} = 0$$

Component	Area (a) mm ²	$\bar{x}$	ax
Total Rect ①	$b \times d = 250 \times 200$ $= 50000$	$\frac{b}{2} = \frac{250}{2} = 125$	6250000
Semicircle ②	$-\frac{\pi R^2}{2} = -\frac{\pi \times 100^2}{2}$ $= -15708$	$\frac{4R}{3\pi} = \frac{4 \times 100}{3\pi}$ $= 42.44$	-666647.52
Alt ③	$\frac{1}{2}bh = \frac{1}{2} \times 200 \times 120$ $= 12000$	$250 + \frac{h}{3}$ $= 250 + \frac{120}{3}$ $= 290$	3480000
$\Sigma a = 46292$		$\Sigma ax =$	9063352.48

$$\bar{x} = \frac{\Sigma ax}{\Sigma a}$$

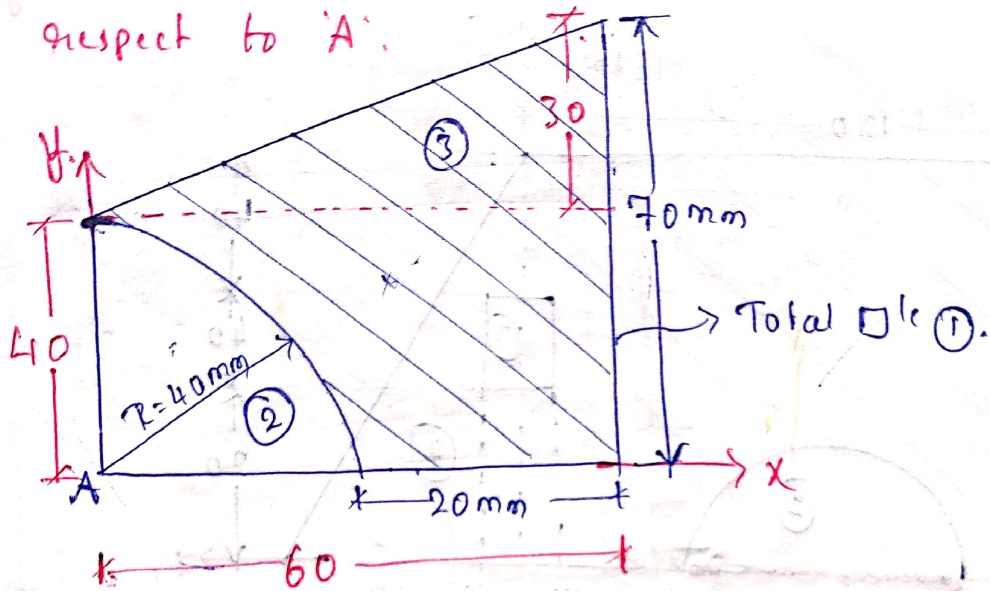
$$= \frac{9063352.48}{46292}$$

$$\bar{x} = 195.78 \text{ mm}$$

$$\bar{y} = 0$$

13) Locate the centroid of lamina shown in Fig. with respect to 'A'.

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Component	Area (a) mm ²	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$a\bar{x}$	$a\bar{y}$
Total $\square$ ①	$(40+20) \times 40$ $= 2400$	$\frac{60}{2} = 30$	$\frac{40}{2} = 20$	72000	48000
Quadrant of the $\odot$ ②	$-\frac{\pi R^2}{4} = -\frac{\pi \times 40^2}{4}$ $= -1256.63$	$\frac{4R}{3\pi} = \frac{4 \times 40}{3\pi}$ $= 16.97$	$\frac{4R}{3\pi} = \frac{4 \times 40}{3\pi}$ $= 16.97$	-21325	-21325
Δ ③	$\frac{1}{2} \times 60 \times (70-40)$ $= 900$	$\frac{2b}{3} = \frac{2 \times 60}{3}$ $= 40$	$40 + \frac{2h}{3} = 40 + \frac{2 \times 30}{3} = 60$	36000	45000
$\Sigma a =$	2043.37			$\Sigma a\bar{x} = 86675$	$\Sigma a\bar{y} = 71675$

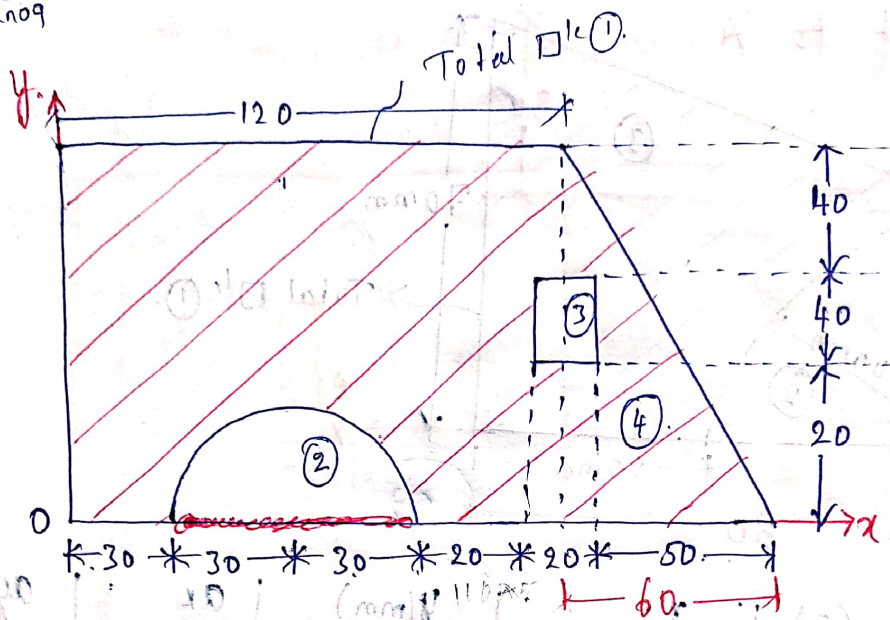
$$\bar{x} = \frac{\Sigma a\bar{x}}{\Sigma a} = \frac{86675}{2043.37} = 42.417 \text{ mm}$$

$$\bar{y} = \frac{\Sigma a\bar{y}}{\Sigma a} = \frac{71675}{2043.37} = 35.07 \text{ mm}$$

14. Locate the centroid of an area shown in fig.

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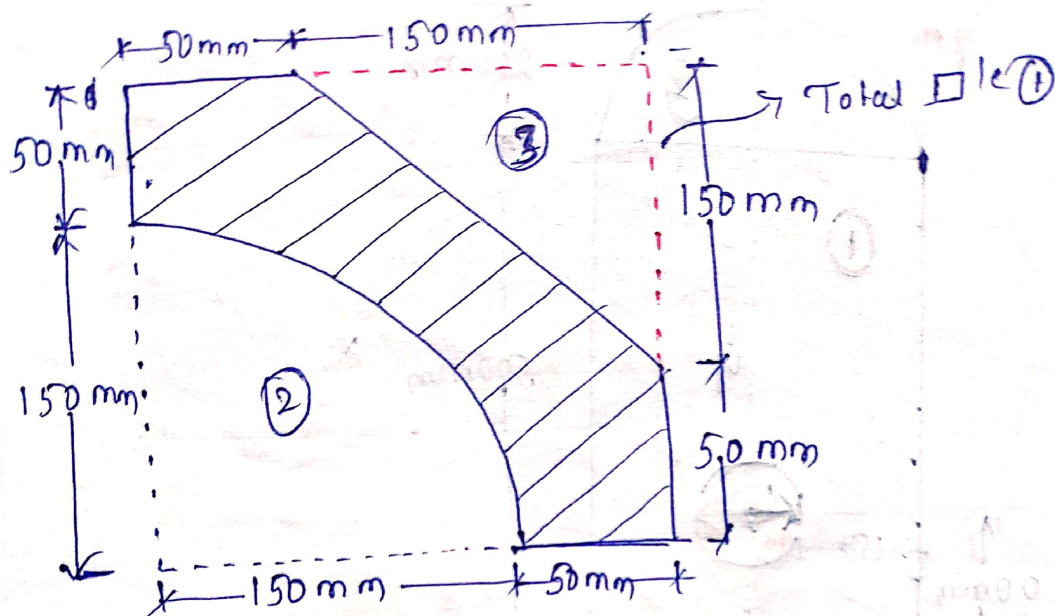


Component	Area (a) mm ²	x (mm)	y (mm)	ax (mm)	ay (mm)
Total Rect ①	$120 \times (40 + 40 + 20)$ $= 12000$	$\frac{b}{2} = \frac{120}{2} = 60$	$\frac{d}{2} = \frac{40 + 40 + 20}{2}$ $= 50$	7200.00	600.000
Semicircle ②	$\frac{\pi R^2}{2} = \frac{\pi \times 30^2}{2}$ $= -1413.7$	$30 + R =$ $30 + 30 = 60$	$\frac{4R}{3\pi} = \frac{4 \times 30}{3\pi}$ $= 12.73$	-848.22	-1799.64
Rect ③	$20 \times 40 = 800$	$30 + 30 + 30 +$ $\frac{20 + 20}{2}$ $= 120$	$20 + \frac{d}{2} =$ $20 + \frac{40}{2} = 40$	-960.00	-320.00
Tri ④	$\frac{1}{2} \times (50 + \frac{50}{2}) \times$ $(20 + 40 + 40)$ $= 3000$	$30 + 30 + 30 + 20$ $\frac{20 + 50}{2}$ $= 140$	$\frac{h}{3} = \frac{40 + 40 + 20}{3}$ $= 33.33$	420.000	999.90
Σa	12786.3			$\Sigma ax = 9591.78$	$\Sigma ay = 649993.6$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{9591.78}{12786.3} = 75.026 \text{ mm}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{649993.6}{12786.3} = 50.84 \text{ mm}$$

15. Determine the centroid of the shaded area shown in fig.



Component	Area (a) mm ²	x	y	a _x	a _y
Total □le ①	$(50+150) \times (150+50) = 40,000$	$\frac{b}{2} = \frac{150+50}{2} = 100$	$\frac{d}{2} = \frac{150+50}{2} = 100$	4000000	4000000
Quadrant of the circle ②	$\frac{\pi R^2}{4} = \frac{\pi \times 150^2}{4} = 17671.45$	$\frac{4R}{3\pi} = \frac{4 \times 150}{3\pi} = 63.66$	$\frac{4R}{3\pi} = \frac{4 \times 150}{3\pi} = 63.66$	-1124964.5	-1124964.5
Δle ③	$\frac{1}{2}bh = \frac{1}{2} \times 150 \times 150 = 11250$	$50 + \frac{2}{3}b = 50 + \frac{2}{3} \times 150 = 150$	$50 + \frac{2}{3} \times 150 = 150$	-1687500	-1687500
Σa	11078.55				

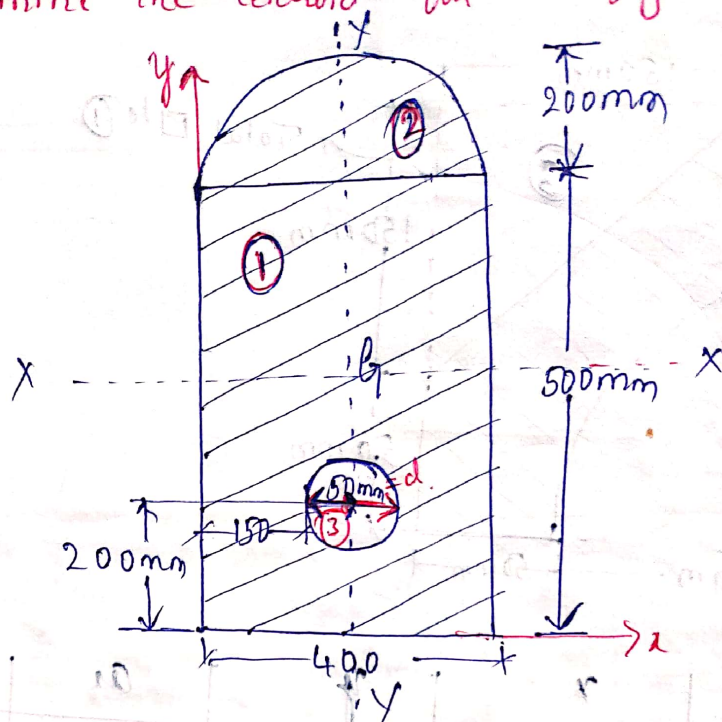
$$\Sigma a_x = 1187535.5$$

$$\Sigma a_y = 1187535.5$$

$$\bar{x} = \frac{\Sigma a_x}{\Sigma a} = \frac{1187535.5}{11078.55} = 107.2 \text{ mm}$$

$$\bar{y} = \frac{\Sigma a_y}{\Sigma a} = 107.2 \text{ mm}$$

16. Determine the centroid for the fig. given below



The given fig. is symmetric about y-axis,

$$\therefore \bar{x} = \frac{400}{2} = 200 \text{ mm.}$$

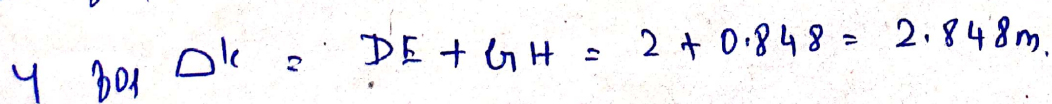
Component	Area (a) mm ²	y (mm)	ay (mm ³)
Rect ①	$400 \times 500 = 200000$	$\frac{d}{2} = \frac{500}{2} = 250$	50000000
Semicircle ②	$\frac{\pi R^2}{2} = \frac{\pi \times 200^2}{2}$ $= 62831.853$	$500 + \frac{4R}{3\pi}$ $= 500 + \frac{4 \times 200}{3\pi}$ $= 584.88$	36749094.18
Hole ③	$-\pi R^2 = -\pi \times 25^2$ $= -1963.75$	200	-392600

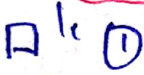
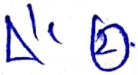
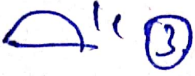
$$\Sigma a = 260868.103$$

$$\Sigma ay = 86356494.18$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{86356494.18}{260868.103}$$

$$\bar{y} = \underline{\underline{331.035 \text{ mm}}}$$

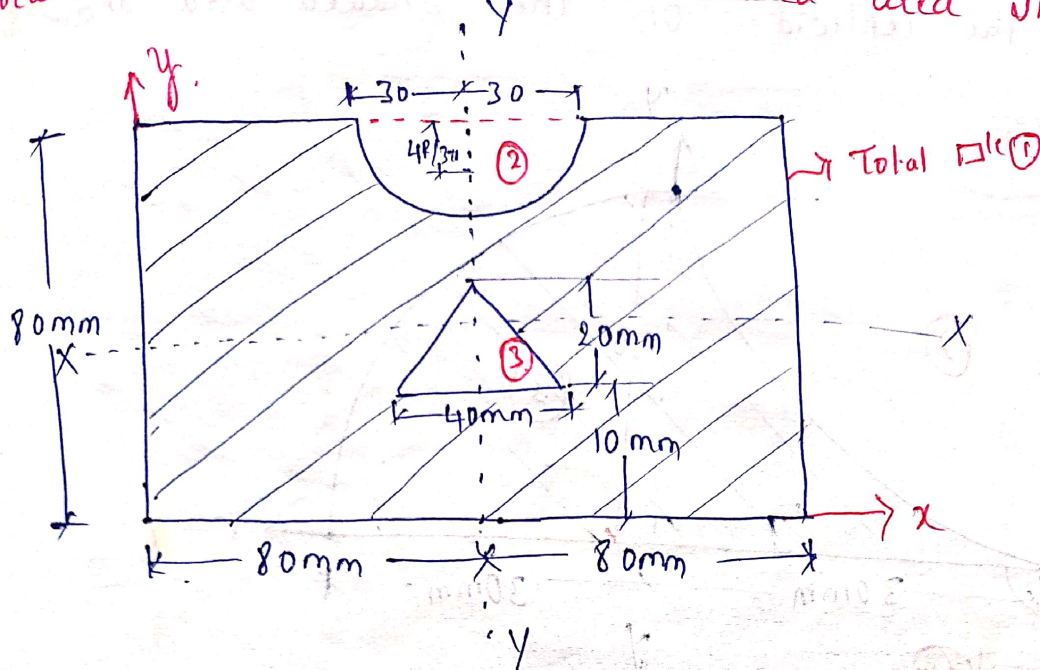


Component	Area (a)	$\bar{x}$	$\bar{y}$	$A\bar{x}$	$A\bar{y}$
 ①	$bd = 4 \times 4 = 16$	$\frac{b}{2} = \frac{4}{2} = 2$	$\frac{d}{2} = \frac{4}{2} = 2$	32	32
 ②	$\frac{1}{2}bh = \frac{1}{2} \times 4 \times 4 = 8$	$4 + \frac{b}{3} = 4 + \frac{4}{3} = 5.33$	$\frac{h}{3} = \frac{4}{3} = 1.33$	42.64	10.64
 ③	$\frac{\pi R^2}{2} = \frac{\pi \times 2.828^2}{2} = 12.56$	6.848	2.848	86	35.77
$\Sigma a = 36.56$				$\Sigma A\bar{x} = 160.44$	$\Sigma A\bar{y} = 78.41$

$$\bar{x} = \frac{\Sigma A\bar{x}}{\Sigma a} = \frac{160.44}{36.56} = 4.389 \text{ m}$$

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma a} = \frac{78.41}{36.56} = 2.145 \text{ m}$$

Locate the centroid of the shaded area shown in fig



The given fig. is symmetric about y-axis
 $\therefore \bar{x} = 80 \text{ mm}$

Component	Area, $a \text{ (mm}^2\text{)}$	y	ay
Rectangle ①	$bd = (80+80) \times 80 = 12800$	$\frac{d}{2} = \frac{80}{2} = 40$	512000
Semicircle ②	$\frac{\pi R^2}{2} = \frac{\pi \times 30^2}{2} = -1413.7$	$80 - \frac{4R}{3\pi} = 80 - \frac{4 \times 30}{3\pi} = 67.26$	-28552.242 -95085.46
Δ^e ③	$-\frac{1}{2}bh = -\frac{1}{2} \times 40 \times 20 = -400$	$10 + \frac{h}{3} = 10 + \frac{20}{3} = 16.66$	-6664
$\Sigma a = 10987$			$\Sigma ay = 410250.54$

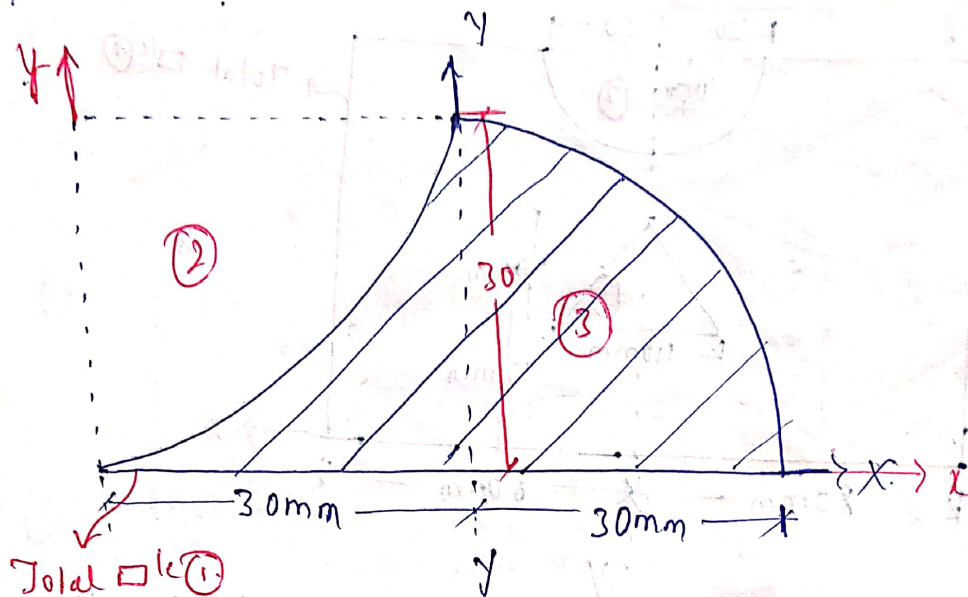
$$\bar{y} = \frac{\Sigma ay}{\Sigma a}$$

$$= \frac{410250.54}{10987}$$

$$\bar{y} = 37.34 \text{ mm}$$

$$\bar{x} = 80 \text{ mm}$$

18. Locate the centroid of the shaded area shown in fig.



Component	Area (a) mm ²	$\bar{x}$	$\bar{y}$	$a\bar{x}$	$a\bar{y}$
Total Rect (1)	$b \times d = 30 \times 30$ $= 900$	$\frac{b}{2} = \frac{30}{2} = 15$	$\frac{d}{2} = \frac{30}{2} = 15$	13500	13500
Quadrant of the circle (2)	$\frac{\pi R^2}{4} = \frac{\pi \times 30^2}{4}$ $= 706.8$	$\frac{4R}{3\pi} = \frac{4 \times 30}{3\pi}$ $= 12.73$	$30 - \frac{4R}{3\pi} = 30 - \frac{4 \times 30}{3\pi}$ $= 17.27$	-8997.56	-12206.4
Quadrant of the circle (3)	$\frac{\pi R^2}{4} = \frac{\pi \times 30^2}{4}$ $= 706.8$	$30 + \frac{4R}{3\pi} = 30 + \frac{4 \times 30}{3\pi}$ $= 42.73 \text{ mm}$	$\frac{4R}{3\pi} = 12.73$	30201.56	8997.56
	$\Sigma a = 900$			$\Sigma a\bar{x} = 34704$	

$$\bar{x} = \frac{\Sigma a\bar{x}}{\Sigma a} = \frac{34704}{900}$$

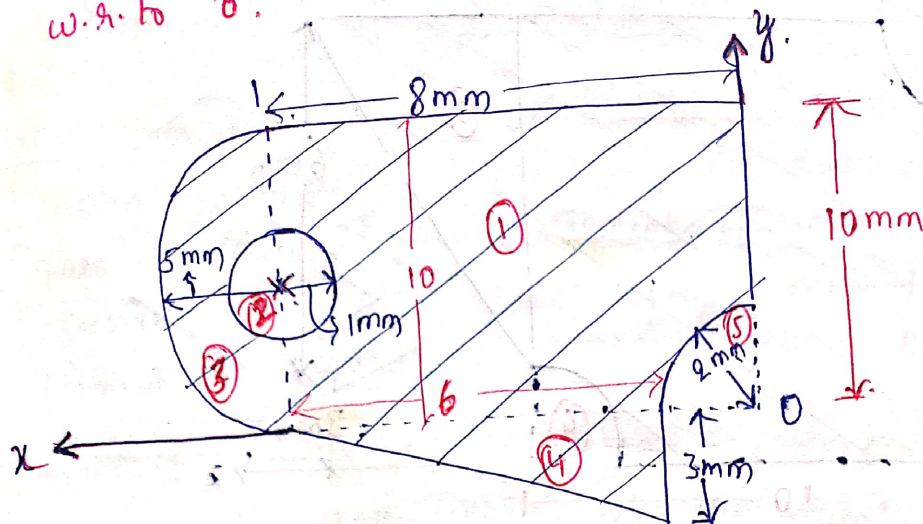
$$\bar{x} = 38.56$$

$$\bar{y} = \frac{\Sigma a\bar{y}}{\Sigma a} = \frac{10291.16}{900}$$

$$\bar{y} = 11.43 \text{ mm}$$

$$\Sigma a\bar{y} = 10291.16$$

19. Locate the centroid of the plane shown in Fig. w.r. to 'O'.

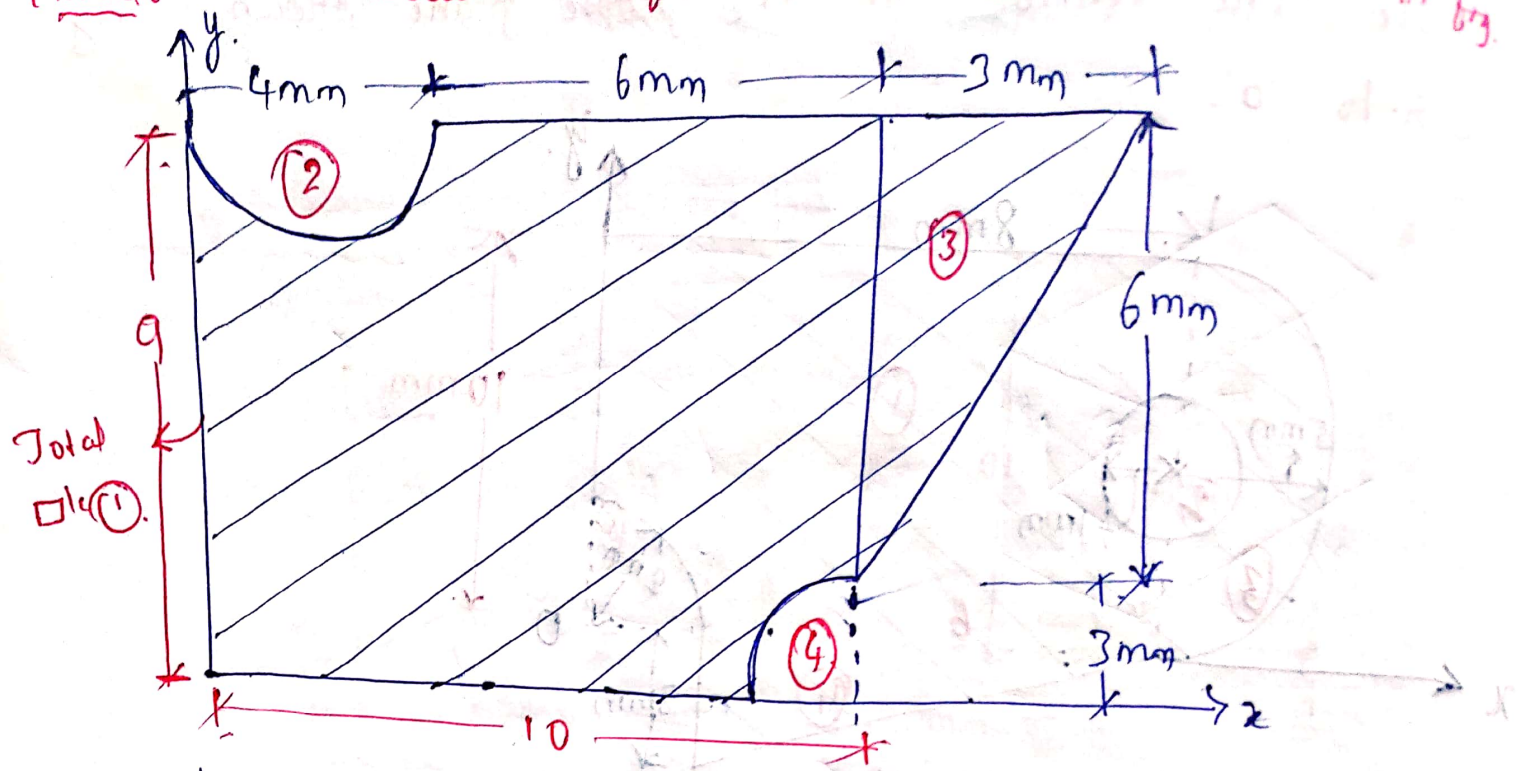


Component	Area 'a' (mm ²)	x	y	ax	ay
Total $\square$ ①	$bd = 8 \times 10 = 80$	$\frac{b}{2} = \frac{8}{2} = 4$	$\frac{d}{2} = \frac{10}{2} = 5$	-320	400
Semicircle ②	$\frac{\pi r^2}{2} = \frac{\pi \times 12}{2} = -3.142$	$\frac{4r}{3\pi} = \frac{4 \times 12}{3\pi} = 16$	$\frac{8r}{3\pi} = \frac{8 \times 12}{3\pi} = 32$	25.136	-15.71
Semicircle ③	$\frac{\pi r^2}{2} = \frac{\pi \times 5^2}{2} = 39.26$	$-(8 + \frac{4r}{3\pi}) = -(8 + \frac{4 \times 5}{3\pi}) = -10.12$	$\frac{8r}{3\pi} = \frac{8 \times 5}{3\pi} = 4.24$	-397.31	+196.3
Δ ④	$\frac{1}{2}bh = \frac{1}{2} \times 6 \times 3 = 9$	$-(2 + \frac{b}{3}) = -(2 + \frac{6}{3}) = -4$	$-\frac{h}{3} = -\frac{3}{3} = -1$	-36	-9
Quadrant of the circle ⑤	$-\frac{\pi r^2}{4} = -\frac{\pi \times 2^2}{4} = -3.142$	$-\frac{4r}{3\pi} = -\frac{4 \times 2}{3\pi} = -0.848$	$\frac{4r}{3\pi} = \frac{4 \times 2}{3\pi} = 0.848$	2.66	-2.66
Σa	121.98			-725.5	$\Sigma ay = 176.33$ 568.93

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{-725.5}{121.98} = -5.94 \text{ mm}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{568.93}{121.98} = 4.664 \text{ mm}$$

2) Locate the centroid of the shaded area shown in fig.



Component	Area	x	y	a_x	a_y
Total □ (1)	$10 \times 9 = 90$	5	4.5	450	405
Δ (2)	$-\frac{\pi R^2}{2} = -\frac{\pi \times 2^2}{2}$ $= -6.28$	2	$9 - \frac{4 \times 2}{3\pi} = 8.151$	-12.56	-51.188
Δ (3)	$\frac{1}{2} \times 3 \times 6 = 9$	$10 + \frac{b}{3} = 10 + \frac{3}{3} = 11$	$3 + \frac{2h}{3} = 3 + \frac{2 \times 6}{3} = 7$	99	63
Δ (4)	$-\frac{\pi R^2}{4} = -\frac{\pi \times 3^2}{4}$ $= -7.06$	$10 - \frac{4 \times 3}{3\pi} = 8.72$	$\frac{4R}{3\pi} = 1.273$	-61.56	-8.98
	$\Sigma a_x = 85.66$			$\Sigma a_x = 474.88$	$\Sigma a_y = 407.83$
	$\bar{x} = \frac{\Sigma a_x}{\Sigma a} = \frac{474.88}{85.66} = 5.543 \text{ mm}$				
	$\bar{y} = \frac{\Sigma a_y}{\Sigma a} = \frac{407.83}{85.66} = 4.76 \text{ mm}$				