

Model Question Paper-II with effect from 2022 (CBCS Scheme)

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Second Semester B.E Degree Examination

Mathematics-II for Computer Science Engineering-BMATS201

TIME: 03 Hours

Max. Marks: 100

- Note:
1. Answer any **FIVE** full questions, choosing at least **ONE** question from each **module**.
 2. VTU Formula Hand Book is permitted.
 3. M: Marks, L: Bloom's level, C: Course outcomes.

Module -1				M	L	C
Q.01	a	Evaluate $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dx dy dz$		7	L2	C01
	b	Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy dy dx$ by changing the order of integration.		7	L3	C01
	c	Show that $\gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$		6	L2	C01
OR						
Q.02	a	Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dx dy$ by changing into polar coordinates		7	L3	C01
	b	Find by double integration, the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.		7	L2	C01
	c	Using Mathematical tools, write the code to find the area of an ellipse by double integration $A = 4 \int_0^a \int_0^{\frac{b}{a}\sqrt{a^2-x^2}} dy dx$.		6	L3	C05
Module-2						
Q. 03	a	Find the directional derivative of $\phi = x^2 yz + 4xz^2$ at $(1, -2, -1)$ in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$.		7	L2	C02
	b	If $\vec{F} = \nabla(xy^3z^2)$, find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ at the point $(1, -1, 1)$		7	L2	C02
	c	Prove that the spherical coordinate system is orthogonal.		6	L2	C02
OR						
Q.04	a	Find the angle between the surfaces $x^2 + y^2 - z^2 = 4$ and $z = x^2 + y^2 - 13$ at $(2, 1, 2)$		7	L3	C02
	b	Show that the vector $\vec{F} = \frac{x\hat{i} + y\hat{j}}{x^2 + y^2}$ is both solenoidal and irrotational.		7	L2	C02
	c	Using Mathematical tools, write the code to find the curl of		6	L3	C05

		$\vec{F} = xy^2\hat{i} + 2x^2yz\hat{j} - 3yz^2\hat{k}$																	
Module-3																			
Q. 05	a	Let $V = \mathbb{R}^3$ be a vector space and consider the subset W of V consisting of vectors of the form (a, a^2, b) , where the second component is the square of the first. Is W a subspace of V ?	7	L2	C03														
	b	Find the basis and the dimension of the subspace spanned by the vectors $\{(2, 4, 2), (1, -1, 0), (1, 2, 1), (0, 3, 1)\}$ in $V_3(\mathbb{R})$.	7	L2	C03														
	c	Find the kernel and range of the linear operator $T(x, y, z) = (x + y, z)$ of $\mathbb{R}^3 \rightarrow \mathbb{R}^2$	6	L2	C03														
OR																			
Q. 06	a	Let $f(x) = 2x^2 - 5$ and $g(x) = x + 1$. Show that the function $h(x) = 4x^2 + 3x - 7$ lies in the subspace $\text{Span}\{f, g\}$ of P_2 .	7	L2	C03														
	b	Prove that the transformation $T; \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(x, y) = (3x, x + y)$ is linear. Find the images of the vectors $(1, 3)$ and $(-1, 2)$ under this transformation.	7	L2	C03														
	c	Show that the functions $f(x) = 3x - 2$ and $g(x) = x$ are orthogonal in P_n with inner product $\langle f, g \rangle = \int_0^1 f(x)g(x)dx$.	6	L2	C03														
Module-4																			
Q. 07	a	Find an approximate value of the root of the equation $xe^x = 3$, using the Regula-Falsi method, carry out three iterations.	7	L2	C04														
	b	Using Newton's divided difference formula, evaluate $f(8)$ from the following <table><tr><td>x</td><td>4</td><td>5</td><td>7</td><td>10</td><td>11</td><td>13</td></tr><tr><td>$f(x)$</td><td>48</td><td>100</td><td>294</td><td>900</td><td>1210</td><td>2028</td></tr></table>	x	4	5	7	10	11	13	$f(x)$	48	100	294	900	1210	2028	7	L2	C04
x	4	5	7	10	11	13													
$f(x)$	48	100	294	900	1210	2028													
	c	Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ using the Trapezoidal rule by taking 6 divisions.	6	L3	C04														
OR																			
Q. 08	a	Find the real root of the equation $\cos x = xe^x$, which is nearer to $x = 0.5$ by the Newton-Raphson method, correct to three decimal places.	7	L2	C04														
	b	Given, $\sin 45^\circ = 0.7071, \sin 50^\circ = 0.7660, \sin 55^\circ = 0.8192,$	7	L2	C04														

		$\sin 60^\circ = 0.8660$, find $\sin 48^\circ$ using Newton's forward interpolation formula.			
	c	Evaluate $\int_0^3 \frac{1}{4x+5} dx$ by using Simpson's 1/3 rd rule by taking 7 ordinates.	6	L3	CO4
Module-5					
Q. 09	a	By Taylor's series method, find the value of y at x = 0.1 and x = 0.2 to 5 places of decimals from $\frac{dy}{dx} = x^2y - 1, y(0) = 1$.	7	L2	CO4
	b	Using the Runge-Kutta method of fourth order, find y(0.1) given that $\frac{dy}{dx} = 3e^x + 2y, y(0) = 0$, taking h = 0.1	7	L2	CO4
	c	Given that $\frac{dy}{dx} = x - y^2$ and the data y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762 compute y at x = 0.8 by applying Milne's method.	6	L2	CO4
OR					
Q. 10	a	Using the modified Euler's method, find y(0.1) given that $\frac{dy}{dx} = x^2 + y$ and y(0) = 1 take step h = 0.05 and perform two modifications in each stage.	7	L2	CO4
	b	Using the Runge-Kutta method of fourth order, find y(0.2) given that $\frac{dy}{dx} = \frac{y-x}{y+x}, y(0.1) = 1.0912$, taking h = 0.1	7	L2	CO4
	c	Using Mathematical tools, write the code to find the solution of $\frac{dy}{dx} = 1 + \frac{y}{x}$ at y(2) taking h = 0.2. Given that y(1) = 2 by Runge-Kutta 4 th order method.	6	L3	CO5