

## Second Semester B.E./B.Tech. Degree Examination, June/July 2024 Mathematics – II for CSE Stream

Time: 3 hrs.

Max. Marks: 100

- Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.  
2. M : Marks, L: Bloom's level, C: Course outcomes.  
3. VTU Hand book is permitted.*

Module – 1			M	L	C
Q.1	a.	Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{(x+y+z)} dz dy dx$ .	7	L2	CO1
	b.	By changing the order of integration evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} xy dy dx$ .	7	L3	CO1
	c.	With usual notation, prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ .	6	L2	CO1
OR					
Q.2	a.	Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} y^2 \sqrt{x^2+y^2} dx dy$ by changing into polar coordinates.	7	L3	CO1
	b.	Find the area bounded between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ by double integration.	7	L2	CO1
	c.	Using Mathematical tool, write the code to find the volume bounded by the sphere $x^2 + y^2 + z^2 = a^2$ by double integration.	6	L3	CO5
Module – 2					
Q.3	a.	Find the directional derivative of $\phi = xy^3 + yz^3$ at the point (2, -1, 1) in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$ .	7	L2	CO2
	b.	Verify whether the vector $\vec{F} = \frac{x\hat{i} + y\hat{j}}{x^2 + y^2}$ is both solenoidal and irrotational.	7	L2	CO2
	c.	Prove that the cylindrical coordinate system is orthogonal.	6	L2	CO2
OR					
Q.4	a.	If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ .	7	L2	CO2
	b.	Find the angle between the normal's to the surface $x^2yz = 1$ at the points (-1, 1, 1) and (1, -1, -1).	7	L3	CO2
	c.	Using mathematical tool write the code to find divergence and curl of the vector $\vec{F} = (4xy - z^3)\hat{i} + 2x^2\hat{j} - 3xz^2\hat{k}$ .	6	L3	CO5



## Module – 3

Q.5	a.	Let $W$ be a subset of $V_3(\mathbb{R})$ consisting of vectors of the form $(a, a^2, b)$ where the second component is the square of the first. Is $W$ a subspace of $V_3(\mathbb{R})$ .	7	L2	CO3
	b.	Let $P_n$ be the vector space of real polynomial functions of degree $\leq n$ . Verify that the transformation $T : P_2 \rightarrow P_1$ defined by $T(ax^2 + bx + c) = (a + b)x + c$ is linear.	7	L2	CO3
	c.	Find the Kernel and range of the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $T(x, y, z) = (x + y, z)$ .	6	L2	CO3

## OR

Q.6	a.	Determine whether or not each of the following $x_1 = (2, 2, 1)$ , $x_2 = (1, 3, 7)$ , $x_3 = (1, 2, 3)$ forms a basis in $\mathbb{R}^3$ .	7	L2	CO3
	b.	Verify Rank-nullity theorem for the transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$ .	7	L2	CO3
	c.	The inner product of the polynomials $f(t) = t + 2$ , $g(t) = 3t - 2$ in $p(t)$ is given by $\langle f, g \rangle = \int_0^1 f(t)g(t) dt$ . Find i) $\langle f, g \rangle$ ii) $\ f\ $ iii) $\ g\ $	6	L2	CO3

## Module – 4

Q.7	a.	Find an approximate root of the equation $\cos x = 3x - 1$ correct to four decimal places using Regula Falsi method between 0.5 and 0.7. <i>2.3061</i>	7	L2	CO4												
	b.	The area 'A' of a circle of diameter 'd' is given by the following table: <table border="1" style="margin: 10px auto;"> <tr> <td>d:</td><td>80</td><td>85</td><td>90</td><td>95</td><td>100</td></tr> <tr> <td>A:</td><td>5026</td><td>5674</td><td>6362</td><td>7088</td><td>7854</td></tr> </table> Using appropriate Newton's interpolation formula for equispaced values of $x$ , find area of the circle corresponding to the diameter 105. <i>8606</i>	d:	80	85	90	95	100	A:	5026	5674	6362	7088	7854	7	L2	CO4
d:	80	85	90	95	100												
A:	5026	5674	6362	7088	7854												
	c.	Evaluate $I = \int_0^5 \frac{1}{4x+5} dx$ by Simpson's $1/3^{\text{rd}}$ rule by considering 10 sub intervals. Hence find an approximate value of $\log 5$ .	6	L3	CO4												

## OR

Q.8	a.	Find the real root of $x \log_{10} x = 1.2$ correct to four decimals that lies near 2.5 using Newton Raphson method.	7	L2	CO4												
	b.	Fit a polynomial for the following data using Newton's divided difference formula: <table border="1" style="margin: 10px auto;"> <tr> <td>x:</td><td>-4</td><td>-1</td><td>0</td><td>2</td><td>5</td></tr> <tr> <td>y:</td><td>1245</td><td>33</td><td>5</td><td>9</td><td>1335</td></tr> </table>	x:	-4	-1	0	2	5	y:	1245	33	5	9	1335	7	L2	CO4
x:	-4	-1	0	2	5												
y:	1245	33	5	9	1335												
	c.	Use trapezoidal rule to find $\int_0^{0.6} e^{-x^2} dx$ by taking seven ordinates.	6	L3	CO4												



## Module – 5

Q.9	a.	Employ Taylor's series method to obtain approximate solution at $x = 0.1$ and $x = 0.2$ for the initial value problem $\frac{dy}{dx} = 2y + 3e^x$ , $y(0) = 0$ . <i>0.3486 0.81</i>	7	L2	CO4
	b.	Apply Runge-Kutta method of fourth order to find an approximate solution at $x = 0.1$ given $\frac{dy}{dx} = 3x + y/2$ , $y(0) = 1$ . <i>1.9236</i>	7	L2	CO4
	c.	Apply Milne's predictor – corrector method to solve the equation $(y^2 + 1)dy - x^2dx = 0$ at $x = 1$ given $y(0) = 1$ , $y(0.25) = 1.0026$ , $y(0.5) = 1.0206$ , $y(0.75) = 1.0679$ .	6	L2	CO4
OR					
Q.10	a.	Apply modified Euler's method to find solution at $x = 0.1$ by taking $h = 0.1$ given $y' = x^2 + y^2$ , $y(0) = 0$ .	7	L2	CO4
	b.	Find an approximate solution of $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ , $y(0) = 1$ at $x = 0.2$ using Runge-Kutta method of order four.	7	L2	CO4
	c.	Write the mathematical tool code to solve $\frac{dy}{dx} = x^2 + y$ , $y(0) = 10$ using Taylor's series method at $x = 0.1(0.1)0.3$ . Consider the terms upto fourth degree.	6	L3	CO5

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