* Inverse transform of logarithmic functions & inverse function

Given $\bar{f}(s)$ we need to find $L^{-1}[\bar{f}(s)] = f(t)$

We have the property $L[t f(t)] = -\bar{f}'(s)$

Equivalently, $L^{-1}[-\bar{f}'(s)] = t f(t)$

⇒ Working procedure for problems:-

step1> In the case of logarithmic functions we apply the properties of logarithmic & then diff. w.r.t s to obtain $\bar{f}'(s)$

step2> We then multiply by -1 & take inverse on both sides

step3> LHS becomes $t f(t)$ by 1 & the inverse are also found for the terms in RHS with the result to obtain the required $f(t)$

step4> If logarithmic function persists in $\bar{f}'(s)$ we diff. again w.r.t s to obtain $\bar{f}''(s)$ & use property that

$L^{-1}[\bar{f}''(s)] = t^2 f(t)$ since $L[t^2 f(t)] = \bar{f}''(s)$

step5> In the cases of inverse function we simply diff. the given $\bar{f}(s)$ & use the result ① to obtain $f(t)$

Ex:-

1) $\log\left(\frac{s+a}{s+b}\right)$

solⁿ- Let $\bar{f}(s) = \log\left(\frac{s+a}{s+b}\right) = \log(s+a) - \log(s+b)$

$-\bar{f}'(s) = -\left[\frac{1}{s+a} - \frac{1}{s+b}\right] = \frac{1}{s+b} - \frac{1}{s+a}$

Now, $L^{-1}[-\bar{f}'(s)] = L^{-1}\left[\frac{1}{s+b}\right] - L^{-1}\left[\frac{1}{s+a}\right]$

$t f(t) = \frac{e^{-bt} - e^{-at}}{t}$

2) $\log\left(1 - \frac{a^2}{s^2}\right)$

solⁿ:- let $\bar{f}(s) = \log\left(1 - \frac{a^2}{s^2}\right) = \log\left(\frac{s^2 - a^2}{s^2}\right)$

$\bar{f}(s) = \log(s^2 - a^2) - 2\log s$

$-\bar{f}'(s) = -\left\{\frac{1}{s^2 - a^2} \times 2s - \frac{2}{s}\right\} = 2\left(\frac{1}{s} - \frac{s}{s^2 - a^2}\right)$

Now, $L^{-1}[-\bar{f}'(s)] = 2\left\{L^{-1}\left(\frac{1}{s}\right) - L^{-1}\left[\frac{s}{s^2 - a^2}\right]\right\}$

$t f(t) = 2(1 - \cosh at)$

$f(t) = \frac{2(1 - \cosh at)}{t}$

3) $\cot^{-1}(s/a)$

soln Let $\bar{f}(s) = \cot^{-1}(s/a)$

diff w.r.t s & multiply with -1

$\bar{f}'(s) = \frac{-1}{1 + (s/a)^2} \times \frac{1}{a}$ & $-\bar{f}'(s) = \frac{a}{s^2 + a^2}$

taking inverse $L^{-1}[-\bar{f}'(s)] = L^{-1}\left(\frac{a}{s^2 + a^2}\right)$

$t f(t) = \sin at$

$f(t) = \sin at / t$

4) $\log\left[\frac{s^2+4}{s(s+4)(s-4)}\right]$

Sol:- Let $\bar{f}(s) = \log\left[\frac{s^2+4}{s(s+4)(s-4)}\right]$

$\bar{f}(s) = \{\log(s^2+4) - \log s - \log(s+4) - \log(s-4)\}$

$\bar{f}'(s) = \frac{2s}{s^2+4} - \frac{1}{s} - \frac{1}{s+4} - \frac{1}{s-4}$

Now. $-L^{-1}[-\bar{f}'(s)] = L^{-1}\left(\frac{1}{s}\right) + L^{-1}\left(\frac{1}{s+4}\right) + L^{-1}\left(\frac{1}{s-4}\right)$
$- 2L^{-1}\left(\frac{s}{s^2+4}\right)$

$t f(t) = 1 + e^{-4t} + e^{4t} - 2\cos 2t$

$= 1 + 2\cosh 4t - 2\cos 2t$

$f(t) = \frac{1 + 2(\cosh 4t - \cos 2t)}{t}$

5) $\tan^{-1}(2/s^2)$

Sol:- Let $\bar{f}(s) = \tan^{-1}(2/s^2)$

$\bar{f}'(s) = \frac{1}{1+(4/s^4)} \cdot \frac{-4}{s^3} = \frac{-4s}{s^4+4}$

$L^{-1}[-\bar{f}'(s)] = L^{-1}\left[\frac{4s}{s^4+4}\right]$ — ①

$t f(t) = L^{-1}\left[\frac{4s}{s^4+4}\right]$

$s^4+4 = (s^2+2)^2 - 4s^2 = (s^2+2+2s)(s^2+2-2s)$

$4s = (s^2+2+2s) - (s^2+2-2s)$

Hence, $\frac{4s}{(s^4+4)} = \frac{(s^2+2+2s) - (s^2+2-2s)}{(s^2+2+2s)(s^2+2-2s)}$

$L^{-1}\left[\frac{4s}{s^4+4}\right] = L^{-1}\left[\frac{1}{s^2-2s+2}\right] - L^{-1}\left[\frac{1}{s^2+2s+2}\right]$

using ① in LHS we have

$t f(t) = L^{-1}\left[\frac{1}{(s-1)^2+1}\right] - L^{-1}\left[\frac{1}{(s+1)^2+1}\right]$

$$t f(t) = e^{t} L^{-1}\left[\frac{1}{s^2+1}\right] - e^{-t} L^{-1}\left[\frac{1}{s^2+1}\right]$$

$$t f(t) = e^{t} L^{-1}\left[\frac{1}{s^2+1}\right]$$

$$t f(t) = e^{t}\sin t - e^{-t}\sin t = \sin t\,(e^{t} - e^{-t})$$

$$t f(t) = \sin t \times 2\sinh t$$

$$f(t) = \frac{2\sin t\,\sinh t}{t}$$

Convolution :-

Definition :- The convolution of two functions $f(t)$ and $g(t)$ usually denoted by $f(t)*g(t)$ is defined in the form of an integral as follows.

$$f(t)*g(t) = \int_{u=0}^{t} f(u)\, g(t-u)\, du$$

property : $f(t)*g(t) = g(t)*f(t)$

Convolution theorem :-

If $L^{-1}[\bar{f}(s)] = f(t)$ and $L^{-1}[\bar{g}(s)] = g(t)$ then

$$L^{-1}[\bar{f}(s)\cdot\bar{g}(s)] = \int_{u=0}^{t} f(u)\, g(t-u)\, du$$

* computation of the inverse transform by using convolution theorem.

⇒ working procedure

step1) The given function is expressed as the product of two functions say $\bar{f}(s)$ & $\bar{g}(s)$

step2) We find $L^{-1}[\bar{f}(s)] = f(t)$ & $L^{-1}[\bar{g}(s)] = g(t)$

3) We apply the convolution theorem in one of the form

$$L^{-1}[\bar{f}(s)\cdot\bar{g}(s)] = \int_{u=0}^{t} f(u)\, g(t-u)\, du$$

4) We evaluate the convolution integral to obtain the required inverse.

⇒ Using convolution theorem to obtain the inverse Laplace transform of the following function.

June 2016 1) 2018

$$\frac{1}{s(s^2+a^2)}$$

Soln:

Let $\bar{f}(s) = \frac{1}{s}$, $\bar{g}(s) = \frac{1}{s^2+a^2}$

$f(t) = L^{-1}\left(\frac{1}{s}\right)$, $g(t) = L^{-1}\left(\frac{1}{s^2+a^2}\right)$

$f(t) = 1$ $\quad g(t) = \frac{\sin at}{a}$

We have convolution theorem.

$$L^{-1}[\bar{f}(s).\bar{g}(s)] = \int_{u=0}^{t} f(u)\, g(t-u)\, du$$

$$L^{-1}\left[\frac{1}{s(s^2-a^2)}\right] = \int_{u=0}^{t} 1. \frac{\sin(at-au)}{a}\, du$$

$$= \left[\frac{\cos(at-au)}{a^2}\right]_0^t$$

$$= \frac{1}{a^2}(1-\cos at)$$

MQP 2)

$$\frac{s}{(s^2+a^2)^2}$$

Soln:

Let $\bar{f}(s) = \frac{1}{s^2+a^2}$, $\bar{g}(s) = \frac{s}{s^2+a^2}$

$f(t) = L^{-1}[\bar{f}(s)]$, $g(t) = L^{-1}\left(\frac{s}{s^2+a^2}\right)$

$= \frac{\sin at}{a}$ $\quad g(t) = \cos at$

Using convolution theorem,

$$L^{-1}[\bar{f}(s) \cdot \bar{g}(s)] = \int_{u=0}^{t} f(u)\, g(t-u)\, du$$

$$L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right] = \int_{u=0}^{t} \frac{\sin au}{a} \cos(at-au)\,du$$

$$= \frac{1}{2a}\int_{u=0}^{t} \sin(au+at-au) + \sin(au-at+au)\,du$$

$$= \frac{1}{2a}\int_{u=0}^{t} [\sin at + \sin(2au-at)\,du$$

$$= \frac{1}{2a}\left[\sin at\,[u]_0^t - \left[\frac{\cos(2au-at)}{2a}\right]_0^t\right.$$

$$= \frac{1}{2a}\left[\sin at(t-0) - \frac{1}{2a}(\cos at - \cos at)\right]$$

$$= \frac{t\sin at}{2a}$$

June 3) 2017 Let $\frac{1}{(s^2+a^2)^2}$

soln:-

$$\bar{f}(s) = \frac{1}{s^2+a^2} = \bar{g}(s)$$

$$f(t) = g(t) = \frac{\sin at}{a}$$

by apply convolution theorem

$$L^{-1}\left[\frac{1}{(s^2+a^2)^2}\right] = \int_{u=0}^{t} \frac{\sin au}{a}\,\frac{\sin(at-au)}{a}\,du$$

$$= \frac{1}{2a^2}\int_{u=0}^{t} [\cos(au-at+au) - \cos(au+at-$$
$$du$$

$$= \frac{1}{2a^2}\int_{u=0}^{t} [\cos(2au-at) - \cos at]\,du$$

$$= \frac{1}{2a^2}\left[\frac{\sin(2au-at)}{2a}\right]_0^t - \cos at\,[u]_0^t\}$$

$$= \frac{1}{2a^3}\left[\frac{\sin at + \sin at}{2}\right\} - \cos at \cdot t\right]$$

$$= \frac{1}{2a^3}[\sin at - at\cos at]$$

4) $\frac{s+2}{(s^2+4s+5)^2}$

Soln- Let $\bar{f}(s) = \frac{s+2}{(s^2+4s+5)}$, $\bar{g}(s) = \frac{1}{(s^2+4s+5)}$

$L^{-1}[\bar{f}(s)] = L^{-1}\left[\frac{(s+2)}{(s+2)^2+1}\right]$, $\bar{g}(s) = L^{-1}\left[\frac{1}{(s+2)^2+1}\right]$

$f(t) = e^{-2t} L^{-1}\left[\frac{s}{s^2+1}\right]$, $g(t) = e^{-2t} L^{-1}\left[\frac{1}{s^2+1}\right]$

$f(t) = e^{-2t}\cos t$, $g(t) = e^{-2t}\sin t$

by applying convolution theorem

$$L^{-1}\left[\frac{(s+2)}{(s^2+4s+5)^2}\right] = \int_{u=0}^{t} e^{-2u}\cos u \cdot e^{-2(t-u)}\sin(t-u)\,du$$

$$= e^{-2t}\int_{u=0}^{t} \sin(t-u)\cos u\, du$$

$$= \frac{e^{-2t}}{2}\int_{u=0}^{t}[\sin(t-u+u)+\sin(t-u-u)]\,du$$

$$= \frac{e^{-2t}}{2}\int_{u=0}^{t}(\sin t+\sin(t-2u))\,du$$

$$= \frac{e^{-2t}}{2}\left[\sin t[u]_0^t + \frac{\cos(t-2u)}{2}\right]_0^t$$

$$= \frac{e^{-2t}}{2}\left[t\sin t + \frac{1}{2}(\cos t-\cos t)\right]$$

$$= \frac{e^{-2t}\, t\sin t}{2}$$

$L[y'(t)] = sL[y(t)] - y(0)$

$L[y''(t)] = s^2 L[y(t)] - sy(0) - y'(0)$

$L[y'''(t)] = s^3 L[y(t)] - s^2 y(0) - sy'(0) - y''(0)$

* Solution of linear differential equation using Laplace transforms.

* Laplace transform of the derivatives.

We derive an expression for $L[y'(t)]$ and hence deduce the expressions for

$L[y''(t)]$, $L[y'''(t)]$

$$L[y'(t)] = \int_0^\infty e^{-st} y'(t)\, dt$$

Integrating by parts we have,

$$L[y'(t)] = [e^{-st} y(t)]_{t=0}^{\infty} - \int_0^\infty y(t) e^{-st}(-s)\, dt$$

$$L[y'(t)] = [0 - 1 . y(0)] + s\int_0^\infty e^{-st} y(t)\, dt$$

$$L[y'(t)] = - y(0) + s L[y(t)]$$

$$L[y'(t)] = sL[y(t)] - y(0) \quad \text{—— (1)}$$

$L[y''(t)] = L[y'(t)]$ & applying (1) we have

$$= sL[y'(t)] - y'(0)$$

$$= s\{sL[y(t)] - y(0)\} - y'(0) \text{ by using (1)}$$

$$L[y''(t)] = s^2 L[y(t)] - sy(0) - y'(0)$$

also, $L[y'''(t)] = s^3 L[y(t)] - s^2 y(0) - y''(0)$

Working procedure

1) The given DE is expressed in the notation $y'(t)$, $y''(t)$, $y'''(t)$ --- for the derivatives.

2) We take LT on both sides of the given eq^n.

3) We use the expression for $L[y'(t)]$, $L[y''(t)]$ ---

4) We substitute the given initial conditions & simplify to obtain $L[y(t)]$ as a function of s

5) We find the inverse to obtain $y(t)$

1) Solve by using Laplace transform: $\frac{d^2y}{dt^2} + k^2 y = 0$ given that $y(0) = 2$, $y'(0) = 0$

Sol:- The given eqⁿ $y''(t) + k^2 y(t) = 0$

taking Laplace transform on b.s we have

$$L[y''(t)] + k^2 L[y(t)] = L(0)$$

$$s^2 L[y(t)] - sy(0) - y'(0)] + k^2 L[y(t)] = 0$$

using the given initial conditions we obtain

$$(s^2 + k^2) L[y(t)] - 2s = 0$$

$$L[y(t)] = \frac{2s}{s^2+k^2}$$

$$y(t) = 2L^{-1}\left[\frac{s}{s^2+k^2}\right] = 2\cos kt$$

2) Solve: $y''' + 2y'' - y' - 2y = 0$, given $y(0) = y'(0) = 0$ & $y''(0) = 6$, by using LT Method.

Sol:- Taking LT on both sides of the given eqⁿ

$$L[y'''(t)] + 2L[y''(t)] - L[y'(t)] - 2L[y(t)] = L(0)$$

$$\{s^3 L[y(t)] - s^2 y(0) - s y'(0) - y''(0)\} + 2[s^2 L[y(t)] - s y(0) - y'(0)] - [s L[y(t)] - y(0)] - 2L[y(t)] = 0$$

using the given initial condition we obtain

$$L[y(t)] [s^3 + 2s^2 - s - 2] - 6 = 0$$

$$L[y(t)] [s^2(s+2) - 1(s+2)] = 6$$

$$L[y(t)] [(s+2)(s^2-1)] = 6$$

$$L[y(t)] = \frac{6}{(s+2)(s-1)(s+1)}$$

$$\frac{6}{(s+2)(s-1)(s+1)} = \frac{A}{(s+2)} + \frac{B}{(s-1)} + \frac{C}{(s+1)}$$

$$6 = A(s-1)(s+1) + B(s+2)(s+1) + C(s+2)(s-1)$$

put $s=-2$, $6 = A(-3)(-1) \Rightarrow A=2$

$s=1$, $6 = B(3)(2) \Rightarrow B=1$

$s=-1$, $6 = C(1)(-2) \Rightarrow C=-3$

hence, $\frac{6}{(s+2)(s-1)(s+1)} = \frac{2}{(s+2)} + \frac{1}{(s-1)} + \frac{-3}{s+1}$

$$L^{-1}\left[\frac{6}{(s+2)(s-1)(s+1)}\right] = 2L^{-1}\left[\frac{1}{s+2}\right] + L^{-1}\left[\frac{1}{s-1}\right] - 3L^{-1}\left[\frac{1}{s+1}\right]$$

$$y(t) = 2e^{-2t} + e^{t} - 3e^{-t}$$

June 2016 3) Solve the following initial value problem by using the LT $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t}$

$y(0)=0$, $y'(0)=0$

soln:- The given eqn is

$$y''(t) + 4y'(t) + 4y(t) = e^{-t}$$

taking LT on bs

$$L[y''(t)] + 4L[y'(t)] + 4L[y(t)] = L[e^{-t}]$$

$$[s^2 L[y(t)] - s\,y(0) - y'(0)] + 4[s\,L[y(t)] - y(0)] + 4L[y(t)] = \frac{1}{s+1}$$

using initial conditions we obtain,

$$L[y(t)][s^2+4s+4] = \frac{1}{s+1}$$

$$L[y(t)] = \frac{1}{(s+1)(s+2)^2}$$

$$y(t) = L^{-1}\left[\frac{1}{(s+1)(s+2)^2}\right]$$

$$\frac{1}{(s+1)(s+2)^2} = \frac{A}{(s+1)} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2}$$

Multiply with $(s+1)(s+2)^2$

$$1 = A(s+2)^2 + B(s+1)(s+2) + C(s+1)$$

put $s=-1$, $A=1$

$s=-2$, $C=-1$

$s=0$, $1=1(4)+B(2)-1(1)$, $B=-1$

Equating coefficient of s^2 on b.s

Hence $\frac{1}{(s+1)(s+2)^2} = \frac{1}{s+1} + \frac{-1}{(s+2)} + \frac{-1}{(s+2)^2}$

$$L^{-1}\left[\frac{1}{(s+1)(s+2)^2}\right] = L^{-1}\left[\frac{1}{s+1}\right] - L^{-1}\left[\frac{1}{(s+2)}\right] - L^{-1}\left[\frac{1}{(s+2)^2}\right]$$

$$y(t) = e^{-t} - e^{-2t} - e^{-2t} L^{-1}\left(\frac{1}{s^2}\right)$$

$$y(t) = e^{t} - e^{-2t} - e^{-2t} t = e^{-t} - (1+t) e^{-2t}$$

June 2017 4) Using LT technique solve: $x'' - 2x' + x = e^{2t}$ with $x(0) = 0$, $x'(0) = -1$

Soln:- Taking LT on bs

$$L[x''(t)] - 2L[x'(t)] + L[x(t)] = L(e^{2t})$$

$$[s^2 L[x(t)] - s x(0) - x'(0)] - 2[s L[x(t)] - x(0)] + L[x(t)] = \frac{1}{s-2}$$

Using the given condition we obtain,

$$[s^2-2s+1] L[x(t)] + 1 = \frac{1}{(s-2)}$$

$$(s-1)^2 L[x(t)] = \frac{1}{s-2} - 1 = \frac{3-s}{s-2}$$

$$L[x(t)] = \frac{3-s}{(s-1)^2(s-2)}$$

$$x(t) = L^{-1}\left[\frac{3-s}{(s-1)^2(s-2)}\right]$$

$$\frac{3-s}{(s-1)^2(s-2)} = \frac{A}{(s-1)} + \frac{B}{(s-1)^2} + \frac{C}{(s-2)}$$

Multiply on bs by $(s-1)^2(s-2)$

$$3-s = A(s-1)(s-2) + B(s-2) + C(s-1)^2 \quad \text{--- (1)}$$

put $s=1$, $2 = B(-1)$, $B = -2$

put $s=2$, $1 = C(1)$, $C = 1$

equating coefficient of s^2 on bs (1)

$$0 = A + C$$

$$A = -1$$

Hence $L^{-1}\left[\frac{3-s}{(s-1)^2(s-2)}\right] = -L^{-1}\left[\frac{1}{s-1}\right] - 2L^{-1}\left[\frac{1}{(s-1)^2}\right] + L^{-1}\left[\frac{1}{(s-2)}\right]$

$$x(t) = -e^t - 2e^t \cdot t + e^{2t} = e^{2t} - (1+2t)e^t$$

MQP 5) solve by using LT on ~~both~~ techniques

$y'' - 3y' + 2y = e^{3t}$, $y(0) = 1$, $y'(0) = 1$

soln:

$y'' - 3y' + 2y = e^{3t}$

$L[y''(t)] - 3L[y'(t)] + 2[L[y(t)]] = L(e^{3t})$

$s^2 L[y(t)] - s\,y(0) - y'(0) - 3\{s L[y(t)] - y(0)\} + 2L[y(t)] = \frac{1}{s-3}$

$L[y(t)](s^2 - 3s + 2) - s(1) - 1 + 3(1) = \frac{1}{s-3}$

$L[y(t)](s^2 - 3s + 2) - s + 2 = \frac{1}{s-3}$

$L[y(t)](s^2 - 3s + 2) = \frac{1}{s-3} + (s-2)$

$= \frac{1 + (s-2)(s-3)}{(s-3)(s-2)(s-1)}$

$L[y(t)] = \frac{1 + s^2 - s - 12}{(s-1)(s-2)(s-3)} = \frac{s^2 - s - 11}{(s-1)(s-2)(s-3)}$

~~dividin~~ Multiplying $(s-1)(s-2)(s-3)$ on b.s

$\frac{s^2 - s - 11}{(s-1)(s-2)(s-3)} = \frac{A}{(s-1)} + \frac{B}{(s-2)} + \frac{C}{(s-3)}$

$s^2 - s - 11 = A(s-2)(s-3) + B(s-1)(s-3) + C(s-1)(s-2)$

put $s = 1 \Rightarrow 1 - 1 - 11 = A(-1)(-2) + 0 + 0$

$-11 = 2A \Rightarrow A = -11/2$

put $s = 2 \Rightarrow 4 - 2 - 11 = 0 + B(1)(-1) + 0$

$-9 = -B \Rightarrow B = 9$

put $s = 3 \Rightarrow 9 - 3 - 11 = 0 + 0 + C(2)(1)$

$-5 = 2C \Rightarrow C = -5/2$

$y(t) = L^{-1}\left[\frac{-11}{2(s-1)}\right] + L^{-1}\left(\frac{9}{s-2}\right) + L^{-1}\left(\frac{-5}{2(s-3)}\right)$

$= \frac{-11}{2}e^{t} + 9e^{2t} - \frac{5}{2}e^{3t}$

$= 1 + s^2 - 5s + 6 = s^2 - 5s + 7$

$s^2 - 5s + 7 = A(s-2)(s-3) + B(s-1)(s-3) + C(s-1)(s-2)$

$s = 1$

$1 - 5 + 7 = A(-1)(-2)$

$3 = 2A \qquad A = 3/2$

MQP 6) solve by using LT techniques

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t \quad \text{with } x(0) = 2 \text{ \& } x'(0) = -1$$

$$x''(t) - 2x'(t) + x = e^t$$

$$L[x''(t)] - 2L[x'(t)] + L[x(t)] = L(e^t)$$

$$s^2 L[x(t)] - s\,x(0) - x'(0) - 2[s L[x(t)] - x(0)]$$
$$+ L[x(t)] = \frac{1}{s-1}$$

$$L[x(t)](s^2 - 2s + 1) - s(2) - (-1) + 2(2) = \frac{1}{s-1}$$
$$-2s + 5 \qquad 4$$

$$L[x(t)](s^2 - 2s + 1) = \frac{1}{s-1} + 2s - 5$$

$$= \frac{1 + 2s^2 - 5s - 2s + 5}{s-1}$$

$$L[x(t)](s^2 - 2s + 1) = \frac{2s^2 - 7s + 6}{(s-1)}$$

$$2s^2 - 7s + 6 = 2s^2 - 4s - 3s + 6 = 0$$
$$2s(s-2) - 3(s-2) = 0$$
$$(s-2)(2s-3)$$

$$s^2 - 2s + 1 = s^2 - s - s + 1 = 0$$
$$s(s-1) - 1(s-1) = 0$$
$$(s-1)(s-1)$$

$$L[x(t)] = \frac{2s^2 - 7s + 6}{(s-1)(s-1)^2}$$

$$\frac{2s^2 - 7s + 6}{(s-1)(s-1)^2} = \frac{A}{(s-1)} + \frac{B}{(s-1)^2} + \frac{C}{(s-1)^3}$$

$$2s^2 - 7s + 6 = A(s-1)^2 + B(s-1) + C$$

put $s = 1$

$$2 - 7 + 6 = 0 + 0 + C$$
$$C = 1$$

Equate the coefficient s^2

$2 = A$

Equate the coefficient of s

$-7 = A(-2) + B + 0$

$-7 = (2)(-2) + B$

$B = -7 + 4 = -3$